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EXISTENCE OF SOLUTIONS FOR QUASILINEAR DELAY INTEGRODIFFERENTIAL EQUATIONS WITH NONLOCAL CONDITIONS

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Abstract . We prove the existence and uniqueness of mild and classical solu - t ion to a quasilinear delay integrodifferential equation with nonlocal condition . The results are obtained by using C_0- semigroup and the Banach fixed point theorem .

1 . Introduction

The existence of solution to evolution equations with nonlocal conditions in Banach space was studied first by Byszewski [6] . In that paper , he established the existence and uniqueness of mild , strong and classical solutions of the nonlocal Cauchy problem

$$u'(t) + Au(t) = f(t, u(t)), t \in (0, a] \tag{1.1}$$

$$u(0) + g(t_1, t_2, ..., t_p, u(t_1), u(t_2)..., u(t_p) = u_0, \tag{1.2}$$

where $0 < t_1 < \cdots < t_p \leq a, -A$ is the infinitesimal generator of a C_0- semigroup in a Banach space $X, u_0 \in X$ and $f : [0, a] \times X \to X, g : [0, a]^p \times X^p \to X$ are given functions . The symbol $g(t_1, ..., t_p, u(\cdot))$ is used in the sense that in the place of " · " we can substitute only elements of the set $(t_1, ..., t_p)$. For example

$$g(t_1, ..., t_p, u(\cdot)) = C_1 u(t_1) + \cdots + C_p u(t_p),$$

where $C_i (i = 1, 2..., p)$ are given constants . Subsequently many authors extended the work to various kind of nonlinear evolution equations [3 , 4 , 7 , 8] .

Several authors have studied the existence of solutions of abstract quasilinear evolution equations in Banach space [1 , 5 , 1 0 , 1 8] . Bahuguna [2] , Oka [1 5] and Oka and Tanaka [1 6] discussed the existence of solutions of quasilinear integrodifferen - tial equations in Banach spaces . Kato [1 2] studied the nonhomogeneous evolution equations and Chandrasekaran [9] proved the existence of mild solutions of the nonlocal Cauchy problem for a nonlinear integrodifferential equation . Dhakne and Pachpatte [1 1] established the existence of a unique strong solution of a quasilinear

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 abstract functional integrodifferential equation in Banach spaces . An equation of
this type o ccurs in a nonlinear conversation law with memory

$$u(t,x)+\Psi(u(t,x))_x=\int_0^t b(t-s)\Psi(u(t,x))_x ds+f(t,x), \quad t\in[0,a], \tag{1.3}$$

$$u(0,x)=\phi(x), \quad x\in\mathbb{R}. \tag{1.4}$$

It is clear that if nonlocal condition (1 . 2) is introduced to (1 . 3) , then it will also have better effect than the classical condition $u(0,x)=\phi(x)$. Therefore , we would like to extend the results for (1 . 1) - (1 . 2) to a class of integrodifferential equations in
Banach spaces .

The aim of this paper is to prove the existence and uniqueness of mild and classical solutions of quasilinear delay integrodifferential equation with nonlocal conditions of the form

$$u'(t)+A(t,u)u(t)=f(t,u(t),u(\alpha(t)))+\int_0^t k(t,s,u(s),u(\beta(s)))ds, \tag{1.5}$$

$$u(0)+g(u)=u_0, \tag{1.6}$$

where $t\in[0,a], A(t,u)$ is the infinitesimal generator of a C_0- semigroup in a Banach

$$\text{space} X, u_0\in X, f:I\times X\times X\to X, k:\Delta\times X\times X\to X, g:C(I:X)\to X,$$

$\alpha,\beta:I\to I$ are given functions . Here $I=[0,a]$ and $\Delta=\{(t,s):0\le s\le t\le a\}$. The results obtained in this paper are generalizations of the results given by Pazy [1 7] , Kato [1 3 , 1 4] and Balachandran and Uchiyama [5] .

2 . Preliminaries

Let X and Y be two Banach spaces such that Y is densely and continuously embedded in X. For any Banach spaces Z the norm of Z is denoted by $\|\cdot\|$ or $\|\cdot\| Z$. The space of all bounded linear operators from X to Y is denoted by $B(X,Y)$ and $B(X,X)$ is written as $B(X)$. We recall some definitions and known facts from
Pazy [1 7] .

Definition 2 . 1 . Let S be a linear operator in X and let Y be a subspace of X. The operator $\tilde{S}$ defined by $D(\tilde{S})=\{x\in D(S)\cap Y : Sx\in Y\}$ and $\tilde{S}_x=Sx$ for $x\in D(\tilde{S})$ is called the part of S in Y.

Definition 2 . 2 . Let B be a subset of X and for every $0\le t\le a$ and $b\in B$, let $A(t,b)$ be the infinitesimal generator of a C_0 semigroup $S_t,b(s),s\ge0$, on X. The family of operators $\{A(t,b)\},(t,b)\in I\times B$, is stable if there are constants $M\ge1$ and ω such that

$$\rho(A(t,b))\supset(\omega,\infty) \quad \text{for}(t,b)\in I\times B,$$

$$\Big\| \prod_{j=1}^{k} R(\lambda:A(t_j,b_j)) \Big\| \le M(\lambda-\omega)^{-k}$$

for $\lambda>\omega$ every finite sequences $0\le t_1\le t_2\le\dots\le t_k\le a, b_j\in B, \quad 1\le j\le k$. The st ability of $\{A(t,b)\},(t,b)\in I\times B$ implies (see [1 7]) that

$$\| \prod_{j=1}^{k} S_{t_j,b_j}(s_j) \| \leq M \exp\{\omega \sum_{j=1}^{k} s_j\}, \quad s_j \geq 0$$

and any finite sequences $0 \leq t_1 \leq t_2 \leq \cdots \leq t_k \leq a, b_j \in B, 1 \leq j \leq k. k = 1, 2, ...$

Definition 2 . 3 . Let $S_{t,b}(s), s \geq 0$ be the C_0- semigroup generatated by $A(t,b)$, $(t,b) \in I \times B$. A subspace Y of X is called $A(t,b)-$ admissible if Y is invariant subspace of $S_{t,b}(s)$ and the restriction of $S_{t,b}(s)$ to Y is a C_0- semigroup in Y.

Let $B \subset X$ be a subset of X such that for every $(t,b) \in I \times B$, $A(t,b)$ is the infinitesimal generator of a C_0- semigroup $S_{t,b}(s), s \geq 0$ on X. We make the following assumptions :

(E 1) The family $\{A(t,b)\}, (t,b) \in I \times B$ is stable .

(E 2) Y is $A(t,b)-$ admissible for $(t,b) \in I \times B$ and the family $\{^{\tilde{A}}(t,b)\}, (t,b) \in I \times B$ of parts $\tilde{A}(t,b)$ of $A(t,b)$ in Y, is stable in Y.

(E 3) For $(t,b) \in I \times B, D(A(t,b)) \supset Y, A(t,b)$ is a bounded linear operator from Y to X and $t \to A(t,b)$ is continuous in the $B(Y,X)$ norm $\| \, . \, \|$ for every

$$b \in B.$$

(E 4) There is a constant $L > 0$ such that

$$\| A(t,b_1) - A(t,b_2) \| Y \to X \quad \leq L \| b_1 - b_2 \| X$$

holds for every $b_1, b_2 \in B$ and $0 \leq t \leq a$.

Let B be a subset of X and $\{A(t,b)\}, (t,b) \in I \times B$ be a family of operators satisfying the conditions (E 1) - (E 4) . If $u \in C(I : X)$ has values in B then there is a unique evolution system $U(t,s;u), 0 \leq s \leq t \leq a$, in X satisfying , (see [1 7 , Theorem 5 . 3 . 1
and Lemma 6 . 4 . 2 , pp . 1 35 , 20 1 - 202]

(i) $\| U(t,s;u) \| \leq Me^{\omega(t-s)}$ for $0 \leq s \leq t \leq a$. where M and ω are stability constants .

(i i) $\frac{\partial^+}{\partial t} U(t,s;u)w = A(s,u(s))U(t,s;u)w$ for $w \in Y$, for $0 \leq s \leq t \leq a$.

(ii i) $\frac{\partial}{\partial s} U(t,s;u)w = -U(t,s;u)A(s,u(s))w$ for $w \in Y$, for $0 \leq s \leq t \leq a$.

Further we assume that

(E 5) For every $u \in C(I : X)$ satisfying $u(t) \in B$ for $0 \leq t \leq a$, we have

$$U(t,s;u)Y \subset Y, \quad 0 \leq s \leq t \leq a$$

and $U(t,s;u)$ is strongly continuous in Y for $0 \leq s \leq t \leq a$. (E 6) Y is reflexive . (E 7) For every $(t,b_1,b_2) \in I \times B \times B, f(t,b_1,b_2) \in Y$. (E 8) $g : C(I : B) \to Y$ is Lipschitz continuous in X and bounded in Y, that is ,

there exist constants $G > 0$ and $G_1 > 0$ such that

$$\| g(u) \| Y \leq G,$$

$$\| g(u) - g(v) \| Y \leq G_1 \max_{t \in I} \| u(t) - v(t) \| X.$$

For the conditions (E 9) and (E 1 0) let Z be taken as both X and Y.

(E 9) $k : \Delta \times Z \to Z$ is continuous and there exist constants $K_1 > 0$ and $K_2 > 0$ such that

$$\int_0^t \| k(t,s,u_1,v_1) - k(t,s,u_2,v_2) \| Z^{ds} \leq K_1(\| u_1(t) - u_2(t) + v_1(t) - v_(t) \| Z),$$

$$K_2 = \max\{\int_0^t \| k(t,s,0,0) \| Zds : (t,s) \in \Delta\}.$$

 (E 10) $f : I \times Z \times Z \to Z$ is continuous and there exist constants $K_3 > 0$ and $K_4 > 0$ such that

$$\| f(t, u_1, v_1) - f(t, u_2, v_2) \| Z \leq K_3(\| u_1 - u_2 \| Z + \| v_1 - v_2 \| Z)$$
$$K_4 = \max_{t \in I} \| f(t, 0, 0) \| Z.$$

Let us take $M_0 = \max \{\| U(t, s; u) \| B(Z) \cdot 0 \leq s \leq t \leq a, u \in B\}$.

(E 11) $\alpha, \beta : I \to I$ is absolutely continuous and there exist constants $b > 0$ and $c > 0$ such that $\alpha'(t) \geq b$ and $\beta'(t) \geq c$ respectively for $t \in I$. (E 1 2)

$$M_0[\| u_0 \| Y + G + r[K_3 a(1 + 1/b) + K_1 a(1 + 1/c)] + a(K_4 + K_2)] \leq r$$
$$q = [Ka \| u_0 \| Y + GKa + M_0 G_1 + M_0[K_3 a(1 + 1/b) + K_1 a(1 + 1/c)]$$
$$+ Ka[r(K_3 a(1 + 1/b) + K_1 a(1 + 1/c))] + a(K_4 + K_2)] < 1.$$

Next we prove the existence of lo cal classical solutions of the quasilinear problem (1 . 5) – (1 . 6) .

For a mild solution of (1 . 5) – (1 . 6) we mean a function $u \in C(I : X)$ with values in B and $u_0 \in X$ satisfying the integral equation

$$u(t) = U^{(t,0\ ;\ u)u_0}_{+\ \int_0^s k(s,\ \tau,} - (^{U(t\ ,\ 0;\ u)g(u)\ +}_{u\ \tau), u(\beta(\tau)))d\tau]ds.} \int_0^t U(t, s; u)[f(s, u(s), u(\alpha(s))) \qquad (2.1)$$

A function $u \in C(I : X)$ such that $u(t) \in D(A(t, u(t)))$ for $t \in (0, a], u \in C^1((0, a] : X)$ and satisfies (1 . 5)) – (1 . 6) in X is called a classical solution of (1 . 5) - (1 . 6) on I. Further there exists a constant $K > 0$ such that for every $u, v \in C(I : X)$ with values in B and every $w \in Y$ we have

$$\| U(t, s; u)w - U(t, s; v)w \| \leq K \| w \| Y \int_s^t \| u(\tau) - v(\tau) \| d\tau. \qquad (2.2)$$

3 . EXISTENCE RESULT **Theorem 3 . 1 .** *Let* $u_0 \in Y$ *and le t* $B = \{u \in X : \| u \| Y \leq r\}, r > 0.$ *If the as - sumptions* (E 1) – (E 1 2) *are satisfied , then* (1 . 5) – (1 . 6) *has a unique classical s o lution*

$$u \in C([0, a] : Y) \cap C^1((0, a] : X)$$

Proof . Let S be a nonempty closed subset of $C([0, a] : X)$ defined by $S = \{u : u \in C([0, a] : X), \| u(t) \| Y \leq r for 0 \leq t \leq a\}$. Consider a mapping P on S defined by

$$(Pu)(t) = U(t, 0; u)u_0 - U(t, 0; u)g(u) + \int_0^t U(t, s; u)[f(s, u(s), u(\alpha(s)))$$
$$+ \int_0^s k(s, \tau, u(\tau), u(\beta(\tau)))d\tau]ds.$$

We claim that P maps S into S. For $u \in S$, we have

$$\| Pu(t) \| Y$$
$$= \| U(t, 0; u)u_0 - U(t, 0; u)g(u) + \int_0^t U(t, s; u)[f(s, u(s), u(\alpha(s)))$$

$$
\begin{aligned}
&+\int_0^s k(s,\tau,u(\tau)u(\beta(\tau)))d\tau]ds \parallel \\
&\leq \parallel U(t,0;u)u_0 \parallel + \parallel U(t,0;u)g(u) \parallel \\
&+\int_0^t \parallel U(t,s;u) \parallel [\parallel f(s,u(s),u(\alpha(s))) - f(s,0,0) \parallel + \parallel f(s,0,0) \parallel \\
&+ \parallel \int_0^s [k(s,\tau,u(\tau),u(\beta(\tau))) - k(s,\tau,0,0)]d\tau \parallel + \parallel \int_0^s k(s,\tau,0,0)d\tau \parallel]ds.
\end{aligned}
$$

Using assumptions (E 8) - (E 1 1) , we get

$$
\begin{aligned}
\parallel Pu(t) \parallel Y &\leq M_0 \parallel u_0 \parallel Y + M_0G + \int_0^t M_0[K_3(\parallel u(s) \parallel + \parallel u(\alpha(s)) \parallel) + K_4 \\
&+\int_0^s K_1(\parallel u(s) \parallel +u(\beta(\tau)) \parallel)d\tau + \int_0^s K_2 d\tau]ds \\
&\leq M_0 \parallel u_0 \parallel Y + M_0G + M_0[K_3ar + K_3 \int_0^t \parallel u(\alpha(s)) \parallel (\alpha'(s)/b)ds \\
&+K_4a + K_1ar + K_1 \int_0^t (\parallel u(\beta(s)) \parallel (\beta'(s)/c)ds + K_2a] \\
&\leq M_0 \parallel u_0 \parallel Y + M_0G + M_0[K_3ar + (K_3/b) \int_{\alpha(0)}^{\alpha(t)} \parallel u(s) \parallel ds + K_4a \\
&+K_1ar + (K_1/c) \int_{\beta(0)}^{\beta(t)} (\parallel u(s) \parallel ds + K_2a] \\
&\leq M_0[\parallel u_0 \parallel Y + G + r[K_3a(1+1/b) + K_1a(1+1/c)] + a(K_4+K_2)]
\end{aligned}
$$

From assumption (E 1 2) , one gets $\parallel Pu(t) \parallel Y \leq r$. Therefore P maps S into it self . Moreover , if $u, v \in S$, then

$$
\begin{aligned}
&\parallel Pu(t) - Pv(t) \parallel \\
&\leq \parallel U(t,0;u)u_0 - U(t,0;v)u_0 \parallel + \parallel U(t,0;u)g(u) - U(t,0;v)g(v) \parallel \\
&+\int_0^t \parallel U(t,s;u)[f(s,u(s),u(\alpha(s))) + \int_0^s k(s,\tau,u(\tau),u(\beta(\tau)))d\tau] \\
&-U(t,s;v)[f(s,v(s),v(\alpha(s))) + \int_0^s k(s,\tau,v(\tau),v((\beta(\tau)))d\tau] \parallel ds \\
&\leq \parallel U(t,0;u)u_0 - U(t,0;v)u_0 \parallel + \parallel U(t,0;u)g(u) - U(t,0;v)g(u) \parallel \\
&- \parallel U(t,0;v)g(u) - U(t,0;v)g(v) \parallel \\
&+\int_0^t \{\parallel U(t,s;u)[f(s,u(s),u(\alpha(s))) + \int_0^s k(s,\tau,u(\tau),u(\beta(\tau)))d\tau] \\
&-U(t,s;v)[f(s,u(s),u(\alpha(s))) + \int_0^s k(s,\tau,u(\tau),u((\beta(\tau)))d\tau] \parallel \\
&+ \parallel U(t,s;v)[f(s,u(s),u(\alpha(s))) + \int_0^s k(s,\tau,u(\tau),u((\beta(\tau)))d\tau] \\
&-U(t,s;v)[f(s,v(s),v(\alpha(s))) + \int_0^s k(s,\tau,v(\tau),v((\beta(\tau)))d\tau] \parallel\}ds
\end{aligned}
$$

 Using assumptions (E 8) - (E 1 2) , one can get

$$
\begin{aligned}
&\| Pu(t) - Pv(t) \| \\
&\leq Ka \| u_0 \| Y \max_{\tau \in I} \| u(\tau) - v(\tau) \| \quad + GKa \max_{\tau \in I} \| u(\tau) - v(\tau) \| \\
&+ M_0 G_1 \max_{\tau \in I} \| u(\tau) - v(\tau) \| \\
&+ Ka \max_{\tau \in I} \| u(\tau) - v(\tau) \| [K_3 \int_0^t \| u(s) \| ds + K_3 \int_0^t \| u(\alpha(s)) \| (\alpha'(s)/b) ds \\
&+ K_4 a + K_1 ar + K_1 \int_0^t \| u(\beta(s)) \| (\beta'(s)/c) ds + K_2 a] \\
&+ M_0 [K_3 \int_0^t \| u(s) - v(s) \| ds + K_3 \int_0^t \| u(\alpha(s)) - v(\alpha(s)) \| (\alpha'(s)/b) ds \\
&+ K_1 a \max_{\tau \in I} \| u(\tau) - v(\tau) \| + K_1 \int_0^t \| u(\beta(s)) - v(\beta(s)) \| (\beta'(s)/c) ds \\
&\leq [Ka \| u_0 \| Y + GKa + M_0 G_1 + M_0 [K_3 a(1 + 1/b) + K_1 a(1 + 1/c)] \\
&+ Ka[r(K_3 a(1 + 1/b) + K_1 a(1 + 1/c))] + a(K_4 + K_2)] \max_{\tau \in I} \| u(\tau) - v(\tau) \| \\
&= q \max_{\tau \in I} \| u(\tau) - v(\tau) \|
\end{aligned}
$$

where $0 < q < 1$. From this inequality it follows that for any $t \in I$,

$$\| Pu(t) - Pv(t) \| \quad \leq q \max_{\tau \in I} \| u(\tau) - v(\tau) \|,$$

so that P is a contraction on S. From the contraction mapping theorem it follows that P has a unique fixed point $u \in S$ which is the mild solution of (1 . 5)) – (1 . 6) on $[0, a]$. Note that $u(t)$ is in $C(I : Y)$ by (E 6) see [1 7 , pp . 1 35 , 201 - 202 Lemma 7 . 4] . In fact , $u(t)$ is weakly continuous as a $Y-$ valued function . This implies that $u(t)$ is separably valued in Y, hence it is strongly measurable . Then $\| u(t) \| Y$ is bounded and measurable function in t. Therefore , $u(t)$ is Bochner integrable (see e . g . [1 9 , Chap . V]) . Using relation $u(t) = Pu(t)$, we conclude that $u(t)$ is in $C(I : Y)$.

Now consider the evolution equation

$$v'(t) + B(t)v(t) = h(t), \quad t \in [0, a] \quad (3.1)$$

$$v(0) = u_0 - g(u) \quad (3.2)$$

$$\text{where} B(t) \quad = A(t, u(t)) \text{and} h(t) \quad = f(t, u(t), u(\alpha(t))) + \int_0^t k(t, s, u(s), u(\beta(s))) ds,$$

$t \in [0, a]$ and u is the unique fixed point of P in S. We note that $B(t)$ satisfies (H 1) - (H 3) in [1 7 , Sec . 5 . 5 . 3] and $h \in C(I : Y)$. Theorem 5 . 5 . 2 of [1 7] implies that there exists a unique function $v \in C(I : Y)$ such that $v \in C^1((0, a], X)$ satisfying (3 . 1) and (3 . 2) in X and v is given by

$$
\begin{aligned}
v(t) = U(t, 0; u)u_0 - U(t, 0; u)g(u) + \int_0^t U(t, s; u)[f(s, u(s), u(\alpha(s))) \\
+ \int_0^s k(s, \tau, u(\tau), u(\beta(\tau))) d\tau] ds,
\end{aligned}
$$

where $U(t, s; u)$ is the evolution system generated by the family $\{A(t, u(t))\}, t \in I$ of the linear operators in X. The uniqueness of v implies that $v = u$ on I and hence u is a unique classical solution of (1 . 5)) – (1 . 6) and $u \in C([0, a] : Y) \cap C^1((0, a] : X)$. □

References

[1] H . Amann , *Quasilinear evolution equations and parabolic systems ,* Trans . Amer . Math . Soc . 29 (1 986) , 1 9 1 - 227 .

[2] D . Bahuguna , *Quasilinear integrodifferential equations in Banach spaces ,* Nonlinear Anal . 24 (1 995) , 1 75 - 183 .

[3] K . Balachandran and M . Chandrasekaran , *Existence of s olution of a delay differential equa - tion with nonlocal condition ,* Indian J . Pure Appl . Math . 27 (1 996) , 443 - 449 .

[4] K . Balachandran and S . Ilamaran , *Existence and uniqueness of mild and strong s olutions of a s emilinear evolution equation with nonlocal conditions ,* Indian J . Pure Appl . Math . 25 (1 994) , 41 1 - 418 .

[5] K . Balachandran and K . Uchiyama , *Existence of s olutions of quasilinear integrodifferential equations with nonlocal condition ,* Tokyo . J . Math . 23 (2000) , 203 - 2 10 .

[6] L . Byszewski , *Theorems about the existence and uniqueness of s olutions of a s emilinear evolution nonlocal Cauchy problem ,* J . Math . Anal . Appl . 1 62 (1 99 1) , 494 - 505 .

[7] L . Byszewski , *Theorems about the existence and uniqueness of continuous s olution of nonlocal problem for nonlinear hyperbolic equation ,* Appl . Anal . 40 (1 99 1) , 173 - 180 .

[8] L . Byszewski , *Uniqueness criterion for so lution to abstract nonlocal Cauchy pro b lem ,* J . Appl . Math . Stoch . Anal . 1 62 (1 991) , 49 - 54 .

[9] M . Chandrasekaran , *Nonlocal Cauchy problem for quasilinear integrodifferential equations in Banach spaces ,* Electron . J . Diff . Eqns . Vol . 2007 (2007) , No . 33 , 1 - 6 . [10] Q . Dong , G . Li and J . Zhang , *Quasilinear nonlocal integrodifferential equations in Banach spaces ,* Electron . J . Diff . Eqns . Vol . 2008 (2008) , No . 1 9 , 1 - 8 . [1 1] M . B . Dhakne and B . G . Pachpatte , *On a quasilinear functional integrodifferential equations in a Banach space ,* Indian J . Pure Appl . Math . 25 (1 994) , 275 - 287 . [1 2] S . Kato , *Nonhomogeneous quasilinear evolution equations in Banach spaces ,* Nonlinear Anal . 9 (1 985) , 1061 - 1071 . [1 3] T . Kato , *Quasilinear equations of evo lution with applications to partial differential equations ,* Lecture Notes in Math . 448 (1 975) , 25 - 70 . [14] T . Kato , *Abstract evolution equation linear and quasilinear ,* revisited , Lecture Notes in Math . 1 540 (1 993) , 103 - 1 25 . [1 5] H . O k − a, *Abstract quasilinear Volterra integrodifferential equations ,* Nonlinear Anal . 28 (1 997) , 10 1 9 - 1045 .
[1 6] H . Oka and N . Tanaka , *Abstract quasilinear integrodifferential equtions of hyperbolic type ,* Nonlinear Anal . 29 (1 997) , 903 - 925 . [17] A . Pazy , *Semigroups of Linear Operators and Applications to Partial Differential Equations ,* Springer , New York (1 983) . [18] N . Sanekata , *Abstract quasilinear equations of evolution in nonreflexive Banach spaces ,* Hi - roshima Mathematical Journal , 1 9 (1 989) , 109 - 1 39 . [1 9] K . Yosida , *Functional Analysis ,* Springer - Verlag , Berlin (1 980) .

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