

[illegible]

o $\mathbb{Q}$ of r a k u
 the M rdel--w el^{grou}p $E(\mathbb{Q})$ Atually, thi i st^he onl y d s a d v^{anta}_{ge}
 ot h Lang--Z ge r_m e ho_d Ho e^{v_{er}} a n a g o r i_h m p o v i d_i g s^{uc} a b
 s s w as r
 c_e n^t l y d v e l o_{pe} d b y t h f i_{r st}^a nd the l a s a u_t or $[G^Z]^I$ t i b a s d o^{nⁱ} d_e
 o f $M_{a i}^n$ [M] a_n d dep n ds o t h e v a l d t o f t h e c o n^e c
 $w-i_n$ r t oⁿ_{Dy} e (s_{ee}[], Z 3], f o r x a m p e). W e_{re} p l a_{nⁿ} n g t o_m a_k
 e i i_n d_{pe} n d e n^f_o t^{hi} c n e^c u e T_h e s c oⁿ_d c o m_p o n e^{t_i}_s a n e x pⁱ i l
 o_w b o^{und}_d f o_r i e a f m_{sin} e l p t c l o^g_{a r} i_h m^s o a^l_{ge} b r a c n u_m b e r s A g a i t
 w_a s o n l y r^{ee}_{n l} y t_h t S D v i d D] e t a b l_{sh} e^{ds} u h a_n e_{x p} i i t b o_{und}
 t h_s p_r vⁱ g a o h_{erco} n_j c t r e o L a n^g_{.T} h s m_e a n t^a b_{rea} t h o^u g h n o
 eⁿ d a v_{or} c_{n^{ce}} n i n iⁿ t g^{ri}_a o i n t s A n^{ai} g o_s e s t m^{a_ites} o l i e r f o r_m i_{nc} o m_{pl} x
 a d^p₋ a d i c^l_{o gar} h m_s h a d b e n s u^{cessu} l_y u s e d f o^r t_h e c o m p e t e e s o^{l_{to}u} n o f t
 h e u, T h u e⁻_M a_{hl} e⁻ r a n_d n d e x f o m e q u a o n_s (e - e_[P_S T d W l, [T d
 W 2, [G P P) T_e r e u c t o n_p r o c e d u e , b a e d o n n^u m_e r c a l d p_h a^{n i n^e}
 a p o x i m t i oⁿ t e c h n q u e_{,is} h t^h_i r i m p o t a n t c o m p o n e n t o o u m_{ethod}
 Wesh all_u s e h e^{rea} v a r i a_n g i v e_{nby} d e W_{ege_r[dW]}
2 . H e g h s . T h e l i p t i c c r v e E v e r $\mathbb{Q}$ s a s s u m e d o b e g i v n i n
 s o r W i e r s r a s i_{sno} r m a l f o r m u o i t e h
 e t 2 3

$$(1) \quad E \quad : \quad y \quad =_x \quad +a \quad x \quad +b =: p(x) \quad ({}^a, {}_b \in \mathbb{Z}).$$

The $d_i c-r_i$ m n-i_a n t of E o v e $\mathbb{Q}$ is

$$\Delta = \quad 4 + \quad 27 \quad \neq 0$$

and the m dul a-r i v r-a i

$$j=12^{34}a3.$$

By th e Morde l - l - We l - i Theo r - e m ,

$o\mathbb{Z}$, w h e and_{edr}

Le t_{u-s} reca_{lt}-l_h e noto no_f hei ht on $E(\mathbb{Q})$. Fo r a rat onalpoint

$$\xi,\zeta)=\gcd(\eta,\zeta)=1$$

h igh t is $\eta \in \mathbb{Z}$, h_e n r t r e { l o m a x { ζ, ξ | } } i f $P \neq O$,

$$h(P) = \quad 02 \quad g \quad | \quad i f P = O \quad ,$$

$$n\rightarrow \infty$$

null piece of h is simply the tor_i -orthogonal group $E_{tor}(\mathbb{Q})$. The $f-e_{ore,h}$ is a $posi^{t-i}$ -valued f -invariant quadratic form

$$E(\mathbb{Q})=E(\mathbb{Q})/E-tor(\mathbb{Q})$$

By embedding $\widehat{E(\mathbb{Q})}$ in the reduced $m_{e_n}s^{ona_{irel}}$ space $E\mathbb{R})\widehat{E(\mathbb{Q})}_{\otimes\mathbb{Z}}\sim\mathbb{R}$,
 to obtain a valid hex -relation to a o_{Euclid^s} e_{nnom} on $E(d_{\mathbb{R}}.i_{Intrhem_E}$ undetermined $a^{\mathbb{R}}$ $ns=pac$ is r

$nkenf$ - geometry of f nu
 Let $P1, P^r \in E(\mathbb{Q})$ denote such a pair (often in n_{i_t} part of $E(\mathbb{Q})$)
 The h_e is a chr -algorithm in P^e $E(\mathbb{Q})$ $a-h_s$ n^{iqu} r^e_{sen} a on off e o^m

$$(2) \quad P = \sum_{i=1}^r i P_i + P_{r+1} \quad (n_i \in \mathbb{Z}),$$

$h_{er-e}P_r+1 \in E_{r-o}(\mathbb{Q})$ is a r_s -nonpoint.
 We want to get rid of $-fP+1$ in (2). To this end, we multiply both sides

$$(2) \quad P = \sum_{i=1}^n niP_i$$

Note that a t -commutative morphism oe^m of M at $z^{ur}[Mz]$, where

$$(3) \quad b^O o^{u^w,e} a - E - n c_{ovep}^r m_{has}^u n^e o t o^n \text{ on ,}$$

f

In order to compute e_{al} in $g_{ra_{poin}}$ s

$E\mathbb{Q})$
 (2) of μP is by the b^nu as points $P^d(i=1,...)$. Put $t n_i$ in t_h e r s e $t a^{to}$

$$) \quad N^i = 1 \leq a_i \leq x_r$$

r -representations (2) $u-d$ lotrion ,

$$P = \sum_n P_i,$$

$i =$

$$(5) \quad (P) \geq \lambda - 1N$$

$$\begin{array}{l} \text{on } \text{on} - t \, r_{si} \, o \, n_p \quad \text{its } P \in E \,) \mathbb{Q}_w \quad \text{e} \quad 0 < \lambda - 1 \in \mathbb{R} \quad \text{hes ample} \\ \text{tg} \quad e^n v_{lu}^a \\ \text{o} \, f_t h_e \quad \text{m} \quad \text{a} \, r_{ix} a s s o c i a \quad \text{d} \, w_t h_a \, \text{n} \, \text{d} \quad \text{the} \, g \, i^{ven} b - a s^{s-i} P 1 \dots, P \quad \text{rof } E(\mathbb{Q}) \end{array}$$

$$\begin{array}{l} \text{Next we} \quad a - r \, e \quad o - g i_n \, g \, t_{or_e} \, \text{plac} \\ \text{o o} \quad \text{nd on } c_E(\mathbb{Q}. \, \text{T} \\ \text{p ont } P \quad = (\xi slash - zeta \quad 2, \eta /) \in E(\mathbb{Q}) s - i \quad \text{defi n d } a_s \text{parenleft - c.} \quad \text{Z 1} \,] - \text{Z three} - i], [\, \text{Zi} \,] \,) \\ \{1 \, \text{m x} \, \{ \infty \quad + 2 \, \text{l g} \, \zeta, \, \text{l g} \, | \, \xi \} \quad \text{if } P \quad \neq \quad O, \end{array}$$

$$dP):= -1\mu \quad \text{if} \quad P=O,$$

$$2 \quad \infty$$

$$\text{with the "height" of } E$$

$$(6) \quad \mu \infty := logmax\{|| \, 12, | \, b \, |^{1/3}\}$$

$$\begin{array}{l} \text{o rth } e^{t_h} f f \quad e o f f i c i e n - t s, \quad \in \mathbb{Z} o f \quad t h e e l l p \quad c c w - e E. \\ r_{een} \quad t a l - b h^s d \end{array}$$

$$\begin{array}{l} (c \quad a_s \quad [^1_{Z_i} \quad] [S2] : \\ (\quad o \quad \leq \quad d_{(P)} - \widetilde{h} P \quad) \leq \quad 2^3 \mu^\infty + \end{array}$$

$$\begin{array}{l} \text{In } f_{act} \quad \text{on } c_m \quad \text{bi } n_{nt}^g \quad h_e \text{ he g t e s}_i m_a t e^s \quad o_t^b \text{ ained i } \quad [Z_2] a n_d \quad [\quad Z \\ w_{th}^i t_h \text{ s e r m } \quad Z \, 5, \text{ one } \quad \text{n d u} \quad \text{pw} \quad t \, h^t \, \text{h} \quad \text{sig} \quad \text{tly} \quad st_o^r n_g \, \text{e} \quad \text{re } \, t^m \quad \text{a} \\ \text{e} \end{array}$$

$$(7) \quad = \frac{7}{1}$$

$$\begin{array}{l} \text{F} \quad \text{i m p l i y} - c o m m a \text{ h o w e e } , e - w \quad h_s l l_a \text{ u s } (7) \quad \text{ra } h_e \\ h_a t_h \text{ e} \quad \text{ei ht } d i^n \quad [Z \quad i \quad - -_Z \, 3^d \quad] \quad i_{ff} \quad \text{rs } f_o^r \quad m_{th} \text{ e } d_d \quad \text{e} \quad f i \, \text{n} \, d_a \quad b^o \, \text{v} \\ \text{e} \quad \text{by a} \quad f^{a_i^c} \, \text{o} \end{array}$$

$$\begin{array}{l} \text{o f . }) \\ \text{e} \quad \wedge \quad 2 \end{array}$$

$$) \quad -3 \quad 2$$

$$\begin{array}{l} \text{H} \quad \text{e n } c_e f_o^r \quad s_u f f i i_e t^{ly} \quad l^a r^{g^e} \quad i \quad n^t e_{gra} \text{ p o i t } s_P \quad = (\, \xi, \eta) \in E \quad (\mathbb{Q}), \text{ m o r e p r e} \\ \text{c i e} \end{array}$$

$$\begin{array}{l}) \\ \text{e lds} \end{array}$$

$$(nine - parenright \quad 12og \, | \, \xi \, | \geq \quad \lambda 1N \quad 2 - 2l_g \quad 2.$$

$$\begin{array}{l} \text{W} \quad \text{er ema rk h a f } \mu \quad \text{slar } g - e \text{ e g ex } \quad \text{p } (\mu) 3 > zero - one 6 \quad \text{nd if } 0 \leq \text{w} \quad \text{m} \\ \text{ate s} \quad ()_7 - - (9) \text{ a s } o_l \text{ low } s. \end{array}$$

$$a_{lp} o^i n s \quad P \in \quad E \quad (\quad \mathbb{Q}) \qquad \qquad \qquad i_{nt} e_r^g$$

$$\text{ower es im a t e } hP) \quad \geq h(P) - \quad \text{e nc}$$

$$(8') \quad 2log|\xi| \geq$$

$$\text{T h} \quad \text{n t h e a e } o_{f0} \leq \text{l o - g } || \leq \mu \quad \infty (9)_i \text{ t o b e r}$$

$$(9' \quad 1 \log |\xi| \geq \lambda N^2 - \quad 2_l g 2 - 1 \mu \quad .$$

$$\text{T h} \quad \text{n} \quad \text{ex r a} \quad e_s \text{ r c} .$$

$$\text{W } ec_o \quad n_{fi} \text{ eo} \quad \text{urel es o} \quad e-x \quad l_a \text{ i n i n} \quad t_g \quad \text{h } s_e \quad \text{rc h} \quad p_rc_e u - d_r \text{ ef}$$

$$o^{r_{larg}} i_n e g_r$$

$$poi_{nt} \quad P = \quad (\xi, \eta$$

$$e-l_{li} \text{ ticl } oga^{i-r} \text{ th m of } P \quad \text{by} \quad \text{u s - i n } gt_h$$

$$\text{cu rve} \quad E(s-e \text{ e } [\text{ L } 2] , \text{ f or exa mple }) \text{ . The re} \quad \text{ex s - it - s a} \quad a-l \text{ ttic}$$

$$\Omega \quad \subseteq \mathbb{C} \quad \text{such} \quad \text{tha}$$

$$\text{the g o } p_u \text{ of} \quad \text{om p } e-x \text{ po nts } s-i$$

$$w_h \text{ e } r_e \quad \Omega \quad = \langle \omega_1 \omega_2 \rangle s_g \text{ n } e_{ra} \text{ ed} \quad b-yt_{wo}f_{un} \text{ d} \quad a^{mental} p_e \text{ r} \quad i_d^o \quad s \quad \omega_1 \text{ an}$$

$$\text{d } \omega_2^o$$

$$\text{w hich } \omega_1 \text{ s re l} \quad \text{a n d } \omega_2 \text{ c} \quad \text{om p}^e x. \text{ We} \quad \text{p t} \quad \tau = \quad 2/\omega_1 \text{ an d s u me w } th_o$$

$$\text{u}$$

$$l_o \text{ s of g e n rat y h } a^t \text{ I m} \quad \tau($$

$$s^p \text{ e} \quad \text{o } \Omega$$

$$P \quad = (\wp parenleft - ucomma - parenright \wp l_{parenleft - u}) \leftarrow u \quad \text{mod } \Omega$$

$$\text{s} \quad \text{oth at h e c oo din es of an int ra po} \quad \text{t } P = (\xi, \quad) \in \quad E(\mathbb{Q}) \quad \text{a r given b t} \quad \text{r}$$

$$\text{a} \quad \text{t} \quad \text{e} \quad \text{l} \quad \text{i n} \quad \eta \quad \text{e}$$

$$\xi = \wp(u), \quad \eta = \wp(u.$$

$$\text{o yn o mial} \quad p - parenleft t^t x) \text{ i n } one - parenleft t parenright - period \text{ The } t_h \text{ (c } , [\text{ Z}$$

$$\text{a } bracketright - parenright$$

$$\omega = 2 \quad \int \quad \sqrt{\quad} \quad x$$

$$\alpha \quad x \quad + \quad a_x + \quad b$$

$$\text{The} \quad \text{eli p t i c lo gar hm o} \quad \text{f } P = \quad parenleft - xi, \eta) \in \quad E \quad (\mathbb{Q}) \text{ is (f . } [\text{ Z a }) \text{ l}$$

$$(11 \quad \omega_1 \xi \quad \sqrt{x} plus - a - x - plus - b 3$$

$$\mathbf{braceleftbt} \quad \sqrt[3]{2} \quad -1$$

$$\mathrm{ad} \quad \mathrm{sa} \quad \mathrm{ea} \quad \{2m_{\alpha+} \quad M \mathrm{In} \quad \mathrm{tel} \, \alpha - a - p - r - \mathrm{comma} \quad \beta - \mathrm{parenleft} \quad + \quad M \quad i - f \\ \mathrm{ic} \quad \log a - e - s_u.t \quad \mathrm{mof} h - te$$

$$(\xi, \quad \mathbb{Q})_{s_{a_t}s} \quad \begin{array}{l} \mathrm{eset} \quad \mathrm{teothi} \\ \mathrm{fi} \quad \xi > \max\{, \} \end{array}$$

$$T \, h e \qquad n \qquad 0$$

$$\int \quad \infty$$

$$4) \quad \sqrt{\quad 3 \quad d} \quad < \sqrt{\quad \xi}.$$

$$\mathrm{n} \, i - t \, \mathrm{must} \quad \mathrm{b} \, \mathrm{e} \, b_o$$

$$O_{th}$$

$$\begin{array}{c} a^{mp_{a-t-e-r-a-n}l}_m \quad \mathrm{Nora} \quad \mathrm{i} \, \mathrm{o} \, cd_{e-e} \\ |\leq 12 \quad a_n \, \mathrm{da} \, \mathrm{s} \quad u-m \quad n-i \quad \mathrm{g} \, \xi > \quad \mathrm{m} \, \mathrm{x} \, \{ \, 0 \, , \, 0 \, \} \quad \mathrm{weobta} \, \mathrm{nf} \, \mathrm{o} \quad \mathrm{m} \quad (\, 11 \\) \, \mathrm{an} \, \mathrm{d} \, (\, 14 \,) \, \mathrm{th} \end{array}$$

$$\mathrm{estimate}$$

$$5 \quad | \, u_{<} \quad \omega 1. \quad | \, \xi \, |.$$

$$\mathrm{O} \, \mathrm{n} \quad \mathrm{cm} \quad \mathrm{bini} \, \mathrm{g} \quad (\, 9 \,) \, \mathrm{an} \mathrm{d} \quad 1 \, 5 \,) \, , \, \mathrm{w} \, \mathrm{e} \, \mathrm{et} \\ \mathrm{o} \quad \mathrm{n} \quad (\quad \sqrt{\quad} \quad \mathrm{g}$$

$$\mathrm{E} \, \mathrm{x} \, 6 \quad / 1_e \quad - \lambda 1 \, \mathrm{fo} \, \mathrm{r}$$

$$c:=2 \quad 2 \cdot \sqrt[4]{\quad}.$$

$$\mathrm{N} \, \mathrm{o} \quad \mathrm{y} \quad \mathrm{t} \, \mathrm{h} \, c^r \, \mathrm{uc} \, \mathrm{i} \, \mathrm{a} \quad T_h e_m^e$$

$$=1n_i i+ur1+$$

$$r\sum_{i=1}$$

$$for some\quad n_i\in e_rn\in\mathbb{Z}. If we replace\quad (2) by (2'), we obtain a orth$$

$$e-l\text{iptl}\quad g\text{rthm}\quad u_i=guo-f\text{th ep }i-o\text{nt }P'=gP\in E(\mathbb{Q})\text{ the represen}$$

$$(18)=\frac{1}{0}+\quad n-prime-ii\quad (n=g^n$$

$$with i-ch\quad weshall\;u_e\;is_t\;ad\;of\;(18)\;.\;Of\;cor-us\;,\;(1_6)i-s\;then\;to\;be$$

$$e-r\;p\;laced\;b$$

$$parenleft-one6\prime)\quad |u|<\quad gcexp-\lambda 1N2\}$$

$$H^e\quad u^m\;e\;te_h\;e\;i-p\;i\;l_g^o\quad t\;h^m\;s\quad n^r\;m\;i-l\;z$$

$$|u_i|$$

$$Since Dvid work sw i tht he c l as s al W ee r s r a s s fo r m$$

$$E\quad:\quad y^2=4x3-\quad gx-$$

$$we\;mu\;str\;ar\;a\;neit\;to\;g\;e\;t$$

$$E:\quad ({}_212)=\quad 3-1_4g2_x\quad -$$

$$s\quad ot\;at\quad we\;ave\quad g=-4a,g\quad=-\quad4. Hence\;,\quad th\;eh\;g$$

$$)$$

$$=h(1,-4a,-4^{b-comma}j$$

$$\times\{\quad4\mid ap,\mid b\mid,p\}\quad+lg\;m\;ax$$

$$p$$

$$e\frac{t}{h}\;es^u\quad m\;m\quad a\;t\quad o\;n$$

$$n\;\mathbb{Q}^z\;W\;it\;in\;gh\;l_{eac}\;m$$

$$\mathbb{Z}a-n\;dus\;n-i\;gth\;e\quad s\;um\quad f\;o\;r\;m\;u\quad l\;a$$

$$p$$

$$weobtainfor\;h\;t\;he\;x\;press\;o-n\;i\quad i$$

$$(02)\quad h=h(\sum-\quad4,-_4b,1j_2)$$

$$x-braceleft0_{lo}\;g\mid4_ap,_{lo}\;g\mid4\mid^p1\quad j^1p-o\mid j_2$$

$$+m\quad a_x0,l^{og|4^a|},$$

$$2_{| \bar{a} - j_1 || j | \},$$

$$\text{eh} \quad \text{x} \, p - \text{one} - r - e \text{ion}^a 2 t_0) a \, \overset{r}{s} \, \text{s t h e v a l u e } 2 a h \, \text{i} \, r_n \quad \text{D a v} \, d - i \, \text{v s y T h e o e} \\ e_{mT} 2 \, \overset{h}{.1} \, . \quad \text{F} \, c - e \, \text{with } (2) := a \, \text{n d } 1_{(2)}, \quad \text{e} \quad \text{a} \quad \text{n} \quad m^{be} \quad \text{s} \quad V 1.. \quad V$$

$$\{ \quad 3\pi \mid \\ (21 \quad \text{o l g} \quad Vi \geq \quad \text{m} \, a^x \quad h(i,) h \quad \omega \, \text{Im} \, (\tau) \quad f_o 1 \leq \quad i \quad \leq r \, \text{a} \quad \text{n} \, , \quad \text{a o r } t - i \, \text{ori} \\ , \quad 1$$

$$(2 \quad 2) \quad B \\ \text{n o} \quad \text{byt} \, h e^b n_e \, \text{nii} \, a - o^y \, \text{n} \, (4o) i_o \, \text{f} \quad o_{seannot}^{N,weha} e - v \quad \text{rth e} \, o_o \, \text{e} \, o_{ffi} \, \text{i t} \, h_e \, \text{e} \, t_i \, \text{m a} \\ \text{ets}$$

$$(4l \quad n-i \leq gN \quad \text{for} 1 \leq i$$

$$\text{i n e r t i n g n } h(6t \text{ th} \text{ , eexp i r s s } eon(18t \text{ for } 8u \text{ w ege } e^e_t s_{ti} \text{ m a e } \quad a_f \text{ o } l^o \quad \text{ws O i} \\ 1 \quad) \quad \textbf{vextendsingle} \quad \text{e r i} \quad \textbf{vextendsingle}$$

$$iequal - one \quad i \quad 1 \\ a_n dt_h \text{ e } r - i \text{ g } t_{ha} n_d \quad i - s_d \text{ e } c_a \text{ n b } e^m \quad a_d \text{ e } \leq 1/2 \text{ f } r_s \text{ uffi c in } t_l \text{ y ar e } N, n_a m_e$$

$$\text{for} \quad \sqrt{\\ N \geq \quad g \lambda 1 \quad c. \\ H - e \text{ n } c_e \quad \text{w} \quad e_o b_{t_a} \text{ n} \\ = i - r - i - vextendsingle - vextendsingle^{n0} \\ \leq \quad 2 +$$

$$\text{a n d}^n \text{ o } N^r \text{ m} > i e e w - o - e - n c h o o^1 \quad) \text{ f } h - t \quad u_{\prime s} \quad \text{n d} \quad \text{b} \\ (\text{parenright} - \text{comma}^{(2)} \cdot \quad \text{h e } f - e - r r^e, \text{ a i} \quad \text{t h} \quad n^B :=) \quad 2 g N_{,o}$$

ma^{t_e(16')} with D₁ a_i^v d's T₁ v in P₁ R₁ op_o SI IO N₁. T₁ he e₁ l_{p-i}
 tc₁ logar₁ t₁ hm₁

$$\sum_r n_0-i=i-r+$$

of an integr al p o int P=(,η)=(ϕ(u) ϕ'(u) ∈E(Q) such that

$$\xi>\max_e$$

$$0\stackrel{a}{s}i)(three-parenrightbigg-parenright(t_i s fit_{(h^e$$

$$\exp\left(-C^h+\log^2gN+\log\log^2gN+1\right)$$

$$\leq |$$

$$where\ N=\max_{i\in\{1,\ldots,n\}}\{|\mathcal{I}_i|\}\leq n-4\},\ then\ one\ can\ find\ \lambda\geq 1$$

$$(\text{ ki } ({}^1g - seven \text{ ar } s - h \stackrel{etv}{s} \text{ dom it i - n g th em } u - d^j c_e \text{ t rm three - l o }$$

$$|g-e-w| \leq \text{co cl u - d C O lar}_{(v} U_{(n \text{ e r } t_{eh}^y)} p^{t^h e)}(s_o \text{ the } p_{(r o p s_i \text{ o })}, \text{)}$$

$$\begin{array}{c} r+1\quad r+1\quad r+1\\ 2\quad 2\quad 2\end{array}$$

$$=1-i$$

Ab o u - a n d e - f - t r - i - n _ { o } ^ { t } e - t ^ { \mathbf { g } } \mathbf { r } - \mathbf { s } \mathbf { l } \mathbf { p } i - \mathbf { o } \mathbf { n } - \mathbf { t } \mathbf { t } \mathbf { s } . Of o - period - cu - r - s - e ^ { e t e - h } o n - e q r - u - e

$$N$$

$$\begin{array}{l} \text{h e - f f i - p c}_{up}os^s, \quad \text{we t fir}^es^r \text{ s tea}^t \text{ o} \\ \text{s L e - m MA period - two L t } \varrho \delta \text{ and } \sigma \text{ erealn mb e - r s sat}_i \\ \text{fyn g} \end{array}$$

$$2-\delta$$

$$\varrho\geq\frac{1}{n},\ \delta\geq\frac{1}{n}\quad and\quad \sigma>\frac{a}{m}x\in\{(e/\delta)^{-1},1\}\quad T\quad en\quad the\quad l-a\quad r\quad et\quad sol\\ i-t\quad n\quad x-zero\in\mathbb{R}\quad of\ he\ e\quad u-q\quad a-t\quad ion\quad x=\varrho+\sigma\log\delta\quad x\ satsfie$$

$$\begin{array}{c} the i-n\\ x0_{<} - 22\varrho\sigma lg(\sigma\delta^{-1}). \end{array}$$

$$\begin{array}{c} : \\ >_0^3\,ud_{greater-zero}^{nd\,1}\quad\text{e um}\quad ec_{o-e-s}^{rt}\text{ n}\quad. \quad U\text{ on}-c\quad so_{i-t-i-d-n-c-h-t}\text{ b}\quad ndo\quad\text{o n w} \end{array}$$

$$\begin{array}{c} 4)\quad \sqrt{()}\leq\quad z32=\sqrt{\xi_-}\alpha. \\ \xi\alpha\quad r\quad z\quad \xi_-\alpha \end{array}$$

$$\text{N o i } \beta \gamma \quad \in \quad \mathbb{R} - \times \gg \supset \text{ we } \quad \text{de ve } \quad \text{rom } \quad \xi/2 \quad > \alpha \quad + M/2 \quad \text{ y } \quad (13) \text{ and } \quad (3) \text{ th t}$$

$$\begin{array}{c} ,\quad \sqrt{\quad}\quad \sqrt{\quad}^3\quad a \\ ich\quad -\frac{l-ds}{\sqrt{\xi}}\quad^t\alpha\quad \sqrt{sse}\quad +_o^M\quad em^{\xi}\quad 1 \end{array}$$

$$\beta+\quad \gamma=\beta+\beta\quad < 2\alpha,th\quad \text{ e } s_{ame}\text{ con u i n } h-o\text{ ld } s_i\text{ n c ea ai n } \xi/2\quad >_{\alpha+}$$

$$\begin{array}{c} \in_{\gamma}^{z\geq}\quad +\quad (+_{nd}^{beta-one-parenright+/\gamma_a}\quad df^{=\beta} \\ (z+\quad)_1\quad z\quad +\beta_1)=z2\quad +\beta\quad 1+\beta1)z+\beta1\beta1 \\ \beta\quad (\quad)2\quad (\quad)2\quad (\quad)2 \end{array}$$

$$\begin{array}{c} 2\quad 2\quad 2 \\ \text{A l g e e } ,_r\text{ o r } z>0(\Leftrightarrow x>\alpha),t-hs-i\quad \text{lea sto the ine } q-u\text{ ali t} \\ \text{n}\quad +\quad \beta_r)(\quad +\quad a^>\quad a+\quad \text{o}\quad 2\quad : \end{array}$$

$$\begin{array}{c} \int^{\infty}\,dz\quad \text{integraltext}-\text{infinity}\quad dz \\ z)\quad \xi-\alpha\quad (+\quad (\beta+\beta1)\quad parenright- three\quad =\quad \xi\quad -\alpha\quad + \\ B_u\text{ t}\quad i_{nt}\text{ h s } c^{as}\text{ e}\quad \text{ si c e by } ({}_13a_n\text{ d }({}_3^3), \end{array}$$

$$>\xi\quad 22_{M_2}^2$$

$$-\alpha \, (1 + \beta \, 1 \, 2$$

$$\text{P r oof}$$

$$\delta_{=}^{x-zero-\delta\!-\!parenleftarrow{rho1-one-slash}<2} \, +^{\sigma} 1\delta_{fi-es}^{\delta\!-\!sigma-\!parenleftarrow{g-o^{\delta}}}\delta_{}$$

$$S_{i_n c_{e\varrho}} \quad \text{and } \sigma a_e^T \text{ at } e^{a_{st_1}} \quad w_e \quad \text{have a n t isimpl est } \varrho_{hea+s-s} r_{\sigma}^e \delta_{ei}^o g_{nequa_{t-i}}^{\sigma \delta}) \stackrel{\leq}{\leq} 2_{y.\varrho} \quad \delta 1^{\delta l}$$

$$g \quad \sigma \delta),$$

$$n_o \text{ S c t i o n 3 we ne ed to } \quad \text{de t e m } i_n \text{ a c o r d n ew i t h t h e c o } \quad \text{d t } o_{ns}(\,$$

$$\omega, \text{ s r s p e c t v e } l-y, t h^t \text{ u g i } i n^E \text{ g } \tau =)^a n_{\omega/} \text{ o } \omega 1 \cdot h_T \, \omega 2, \text{ w e c h o o s e } \quad \text{oro ur elipt c}$$

$$cu^{r-v} \quad \text{e } t h_e$$

$$v_2=r(\quad)=z(z+\quad\beta_1(z\quad+\gamma 1)$$

$$\text{a nd apply th e m e h } \quad \text{o do farith } \quad \text{m t i c - g o } \quad \text{m e r i c } \quad \text{m e a n } \quad \text{o f G a u s a s d e}$$

$$\text{sc ibe by Gr ayson } \quad \text{Gr} \, . \quad \text{F r } \quad \text{h } \quad \text{e c - o } \quad \text{m p u a t o o f t h e e l p t c } \quad \text{o g a r i h m s}$$

$$uif-o \text{ th p o i n s } P \quad (1 \leq i \quad \leq r) \text{ we u } \quad \text{s e t h e f a } t^i - nonv \text{ e g i n s - e r i e } l_g \text{ venby}$$

$$\text{Z a g e } \quad [\text{Z f o m u a } \quad (10) \quad \text{O f o u } \quad \text{se , t h e N e r o n T } s_a \text{ e h e i g h t f t h e b}$$

$$\text{d } \quad \text{n (21)} \stackrel{s}{i} c \text{ a c - u a - l t e d b y t h}$$

$$\text{a e } ad^y \text{ u s e d i } [\text{ G } \quad \text{Z}$$

$$\mathbf{6} \quad \mathbf{R} \quad \mathbf{e^d u t c i n o f t h e i} \quad \mathbf{n_t a_1 b o u} \quad \mathbf{nd} \, . \quad \text{T } \quad \text{h u } p^p \quad \text{e } \quad b_{oun} d_f \text{ r } \quad N^o \text{ b } t^a \text{ i n}$$

$$\text{int } h_e \text{ T } \quad \text{h e } \quad \text{rm } \quad \text{isin } \quad g \text{ e}_n \text{ r } l t o^o l_{a r_g} \text{ f } \quad o_r^{c_o} \quad \text{m p } u_t n_g a_l i^{n^t e g r} a_p \quad \text{s o u e l}$$

$$\text{p } i_c \quad \text{u } r^v \text{ E o v e } \quad r^{\mathbb{Q}} \quad . \quad H_o \text{ w e v e , } \quad \text{b } \quad \text{y } n_u \text{ m e i a } \quad d_i o_{phan} n^{e a} \quad \text{p } p_{rox} \text{ m}$$

$$\text{a } \quad t_o \text{ n t e c } h n i q^{ue}$$

$$e \quad u_l t_y$$

$$\text{vextendsingle/ } \sum \quad \text{vextendsingle vextendsingle } \quad / \quad 2$$

$$\text{vextendsinglen}0 + \quad \nu i \text{vextendsingle} < \quad g_c \exp -^{\lambda} \quad 1N\}$$

$$i = 1 \quad / \quad / \quad /$$

$$(35)$$

$$\text{o b t a i } n_e \quad d i_n h^e \text{ P r o p o s t i o n f o } r_u \quad = \quad g_b^u \text{ y } v^i \quad r_u^t \text{ e o f } (1 \quad 6_a) \text{ n d } (18)^{t o} \text{ e t - } h_e$$

$$\text{w t } \quad \text{h t h e}$$

$$\text{es t } \quad \text{Th o e - } m_e \quad \text{m m a y } \quad \text{e - b reg arde dasa } \quad \text{h of } \quad \text{expo ne n t - } i_a l d_{o-i} \text{ p}$$

$$\text{h a n t n e e u } \quad a_t o - n^s, \text{ a n d } \quad \text{m e h o d s } \quad \text{f o r s o v i n } g_t \text{ h e } \quad \text{m h a v}$$

$$7\text{ . E x a m p l e s } \text{EX A MPE } 1y2 = \text{ } x3 - 1 \text{ } 620three - twox \text{ } + 62877920 \text{ L } 4$$

$$\frac{\text{We a } k_e}{r - a_{sso - frm} \text{ e a}}$$

$$\begin{array}{l} E \quad : \quad y2 = x3 - 1642032x + \\ \Delta \quad = 11two - five7102923^7 7_{94}28_35^2 \quad =^2 \quad 2^3 \quad \quad \quad .511_445_075_7, \\ \text{mo d ula } r - i \end{array}$$

$$\begin{array}{l} = j^1 = \quad 224933_{one - nine}718496 \\ \text{h o f } E \quad 1/ \quad 2 \quad /3 \end{array}$$

$$\begin{array}{l} \mu \\ odebw_{\mathfrak{l}}^{Ep} \left(\mathbb{Q} \right) = \{O\}, \quad sothofMmabasisofth \\ W \quad a_{\mathfrak{l}s} \text{ o } \quad \text{dis p a y} \end{array}$$

$$\begin{array}{l} P1 =_{(\quad 432108), \quad} \widehat{h} \quad (P_1) =_3 \quad .363710642_5 \\ P \quad 2 =_{\mathfrak{6}}^{(39} \quad 6_{3,7}2), \quad \mathfrak{h} - parenleft^P 2) = \quad 33_88408_52_9, \\ P_3 = (360,980), \quad \widehat{h}(P \quad) \quad = 3one - four293916two - zero, \quad \widehat{\quad} \end{array}$$

$$\begin{array}{l} P = \quad (108 \quad 212 \quad \widehat{h}(P \quad) \quad 3. \quad 559132453 \\ P = \quad (362 \quad 3868 \quad \widehat{h}(P \quad) \quad 3 \quad .595291970 \end{array}$$

$$\begin{array}{l} \text{tr c m atix } \quad \text{o fth e } \quad \text{n ear f r m a s ociatd } \quad \text{w} \\ \text{ } \quad \quad \quad \text{i} \\ h - a_bw^ni - o \text{ f ac h T z a nak i } \quad \text{p } \quad a^{p - e} \text{ s } \quad comma - op^e \text{ o r ed d o nanag } \quad a_o \\ \text{ } \quad \quad \quad \text{a} \\ e_{\mathfrak{l}}p^t \text{ i } \quad cc_ar - v \text{ esby m } \quad \text{s oe l i } \quad t^i \text{ l } \quad g - o \text{ arthms } \quad .Af_t \\ \text{im } \quad \text{w } \quad \text{e } \quad \text{h } \quad \text{p } \quad p^i n^t \quad \text{o } \quad \text{a } \quad \text{T } \quad \text{n k } \quad . \end{array}$$

$$\begin{aligned}
& \hspace{10em} 3_4 1_2 9 \quad 1 \\
& \hspace{2em} -12380^1 6 \quad 8617 \quad zero-period_6 1460 \quad - .2_1 851 \quad - 1576 \\
A-e-t v^{x_x}) a^{A=h6^x_a}_{l \ u \ _{eso}+} \ h e^{c.8501 har}_{25^A-2_0} \cdot are 2937 \quad + \frac{-90}{t_0^{r-e} i_5^{s-t} x_{3x} \ 2ic} \quad 1429^{2386}_{5^{y_4}_{8^2} nom_9 \ a-i} \quad +^{7x^4} 226^{-61}_{.2} \quad 7^{763540}_{638.} x \\
& \hspace{10em} \lambda 1 = 0.432372401, \quad \lambda 4 = 4.
\end{aligned}$$

$$\lambda 2 = 1.564757846 \quad \lambda 5 = \quad 5period-five147069_9,$$

$$\lambda 3 \quad = 1.9944767seven-nine, \quad \lambda 6 \quad = 7_{zero-period} zero-six831_1 531,$$

$$\begin{aligned}
& \text{of which } \lambda 1 \quad i-s \quad ne \ d \ d \ i \\
& \hspace{2em} r_o \ d \ i \ v n \quad \text{by } (\ 1 \ 0 \ i
\end{aligned}$$

$$\omega 1 = 1 \quad 8_2 6^7 843$$

$$\text{xp } r_e \text{ o dis tha t}$$

$$= \omega^2 = \quad 0.3843290$$

$$\begin{aligned}
& \text{He nce thecons an tin } (\ 1 \\
& \hspace{10em} 1
\end{aligned}$$

$$\begin{aligned}
& \hspace{10em} 1_\lambda \quad , \quad 1 \quad = \quad 1^4 \\
I_n \quad (1)_w \text{ en} \quad e \ e_{dt} \ h \quad ee \quad l^{ip} \ t \quad l \quad ogath \ m^s \quad o \ t \ e \quad bass \quad p \ o \ i \ t \ s_{P1 \cdot \cdot}^P_{6:}
\end{aligned}$$

$$\begin{aligned}
& \hspace{10em} 4_{95}8842^3, \\
u = 0.zero-zero-nine-six \quad , \quad u-u = 0_02047900793_2 \\
& \hspace{10em} 0_3
\end{aligned}$$

$$\begin{aligned}
& \hspace{10em} \text{a nd} \quad \text{t eq uant tyf o} \quad \text{m } (\ 20 \) \ , \\
\text{lo m a } x-braceleft \quad bar-fouraj2bar-comma4 \mid b_j \quad j_1j,2_{\mid} \} = \quad 4period-six3145_3 \\
& \hspace{10em} \wedge \quad 3\pi u_i \\
& \hspace{10em} i
\end{aligned}$$

$$\begin{aligned}
&) \quad \tau \text{ and } (2^2). \text{ It } \quad t u^r n^s \quad \text{ou tth} \quad \text{a ll } C_{\sim} \quad 7 \cdot 10 \quad \text{a n} \quad t_{there} \\
& \quad \text{m xa} \quad C109 \quad h \quad r+1 \quad \prod \log Vi = \frac{2}{h} +_1 \quad \sim \quad 2.5 \\
& \quad \lambda \quad 2 \quad = 1 \quad 2 \quad \lambda \\
& \quad \text{F i nal } l-y \quad \text{i n } (\quad 1 \quad) \quad \text{w g } e_t \quad N1 = 1.11012 \\
& \quad \text{d} \quad \text{eTh o e} - m \quad s^h w - N s - l e s s e q u a l^t a - N \text{ t} \quad \text{m} \quad \text{a} \quad h7 \} \quad = \\
& \quad \text{Now} \quad \text{w ea} \quad \text{p p y de} \quad \text{We} \\
& \quad \quad \quad 0 \quad 0 \\
& \quad \text{parenleftex} \quad 0 \quad 1 \\
& \quad \quad \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \\
& \quad \text{parenleftexparenleftex} \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \\
& -n-nC0 = {}^{i-w^C_{e^c_{t-h}}}_2 \quad 0^{one-two^{v-a^s_{i-l}}_C} \quad \text{toe} \quad \text{uretha}^{t^{C=N^b}} \quad (C_{l^w-e}36) \text{ is } \quad f i^c_C t s f i e d. \quad C_{1^c_{f_i}} r e d u c \text{ e } \quad C_{0^t_u} h e s e v e n - t - l - z e r o - e - t \\
& \quad \quad \quad t_h \text{ e s h e r t s } - t \ve e_c \text{ or } b_1 \quad f_t \quad e^h \quad \text{a t t i c} \quad \text{t} \quad \| b_1 \quad \geq 114 s e v e n - p e r i o d \quad \text{n} \\
& \quad \text{d y} \quad \text{ie } d_s t^h \quad e_b \quad \text{o } u_n \\
& \quad \quad \quad N \leq 10 \\
& \quad \quad \quad \text{A } t h_i \text{ d r e d } u c^{io} 1 \quad \text{e d s o h } e s_a \\
& \quad d_u \text{ t o n} \quad \text{o n e} \quad c^{to_{rs}} o_{f_r} \text{ e a} \quad n_u m_b e - r_{s n} \text{ s e d } - a \quad o_{i n}^f \text{ g r a } l_\nu \text{ m b e }_r \text{ s a }_s i^n \\
& \quad \text{d} \\
& \quad \quad \text{W} \quad \text{e e} - r, \text{ m e h o d}, \quad h_e \quad \text{upper b o}_{u n} \text{ d i s i } m_p \text{ r o v e d}_t \text{ o } \quad N \quad \leq 8.7 \text{ h s} \\
& \quad i_m p_r \quad o^{v m} \text{ e n} \\
& \quad \quad i_s \text{ a } c h i e_{v e} d \sin c_e, \text{ y} \quad \text{p a} - s \text{ s } n - i_g \text{ t o } t_e^h \quad r_e \quad \text{l c a s} \quad , \quad \text{n e o b a i n} - s \text{ a s} \\
& \quad h a_r \text{ p e }_r s t i m t - a \text{ f o t } h_e \\
& \quad \quad \text{O} \quad \text{ur} \quad \text{t s } k_s t_{e e}^h f_{o r e_t} o_t \text{ e s } - t \quad \text{or nt g r a } i^t y_{a l} \quad h - t \text{ e p o } - i - n_t \text{ s} \\
& \quad \quad \quad P \quad = n P \quad 1 + \quad n 2 P 2 + \quad n 3 P 3 + \quad n P 4 \quad + \quad n_5 P 5 \quad + n 6 P 6 \\
& \quad \quad \quad {}_4 \text{ s u } c^h \quad \text{th a} \quad |n_i| \quad \leq \quad N^{\leq} \quad 8(1 \quad \leq_i \leq \quad 6). \text{ It } \quad \text{the} \quad r_e \text{ m a} \quad \text{i s t o} \quad \text{t } e^{s t} \\
& \quad \text{th e po } i_n \\
& \quad \quad \quad P \quad =_{(\xi, \eta) \in} E(\mathbb{Q})_{s u c h} \text{ t h a t} \\
& \quad \quad \quad \xi \in \quad \mathbb{Z} \text{ a n d } \quad 0 \quad \leq \log | \xi | \quad \leq \mu \infty = 7.15722 \quad 52
\end{aligned}$$

$$8) \quad 0 \quad 0 \quad 1 \quad 4 \, 99 \, 1 \quad 1$$

zero – five – five – eight – nine – nine – one – two – seven – five – two – two – three – three – four – f

$$o_{mM_e} \text{ sr} \left[^M \text{ e o } f^r \text{ a k } \right] r = 5 \text{ o e r } \mathbb{Q} \text{ a n d}$$

form ab a isof

$$N1=1\cdot 10^3 \text{ and } C=0=0600 \text{ w e}$$

$$\frac{tupo_p}{n-dto}N\leq sr_am-e.$$

$$W_{eits} \text{ h e } firs_{c o o r d} \text{ i n } e \text{ s } \xi_h^{o f t}(2 \times) e_e \text{ r } s_{r a s}^t s^m \text{ o d } l_o \text{ f } E \text{ v e r } \mathbb{Q} \text{ w h i h}$$
$$c_o s i t^{u t e} e \text{ e } t_o f_a \text{ i } n_{t_{e g}} r l_{o i n t}$$

$$-3_{six-comma} \text{ comma-four three-six two-seven, } 8one-comma8comma-eight39, \text{ four-three}2, -4_{88}, -68, -_4 \text{ } 3_2, -43, -$$

$$9\,108\,1\,572\,,163\,2\,,13\,8\,76\,,5\,269\,,4\,5\,5\,8\,3\,6\,,7\,29\,r\,6\,.$$
$$\text{Mes re [M e] co m put so n yt h e } (2 \times) \text{ } 31 \text{ n egra}$$

$$1\,0\,nth\,m\,2\,d\,l\,i\,n\,l\,n\,3\,g\,W\,e\,r\,s\,2\,r\,s\,f\,o\,r\,m$$

$$+67=x-21x-10x+30$$
$$\frac{o\,h\,m\,d\,n^a\,m}{1^6523,-1431489_9}\,a_n^d\,(10594$$

$$=x-87^99^8x+3\,1387_0four-period$$
$$W\,e\,t\,a\,e_t\,h\,e\,ur_v\,e$$

$$E:\,y^2$$
$$k\,r=5\,o\,v\,e\,r\,\mathbb{Q}.\,\,\,\mathrm{T}\,\,\,\mathrm{hepoi}$$

$$(54zero-commal1eight-eight),_{(571836)},\,\,\,parenleft-fouright-six,3132parenright-comma\,\,\,(6_1$$

$$N1=^2\,1\cdot0^9\,\text{a n }\text{d } C0=10600\,\,\,\text{wob r noim p ov e ment A s n ab }v-e,e\,\text{b }t_y$$
$$eim_p\text{ r oved o }N\leq6.$$

Ta l e 3

$3,1_{0,1}7,376,3\quad 9_{,43}two-comma46comma-eight5_13,52,50,57^{6,5}8\cdot61_2\quad 6,$

$seven-two0,792,972,172,138,\quad 1732,211comma-three^2385\quad 2four-eight,25_9$

$\quad 2\quad ,\quad 368\quad 8$

$\quad \quad \quad \mathbf{R}\quad \mathbf{e}\quad \mathbf{een}\quad \mathbf{es}$

] A . B a ker , \quad T e \quad Diop h n tine \quad e uaion \quad y \quad 2 = ax3 + bx2 + \quad cx + d, J . Lon d

$c_cu_rve:re$

$1_{--}3.$

$I\quad Ga^{\mathcal{A}}.P\quad e_t\quad h$

$i_{uad}\quad r\quad c_u^n\quad mb\quad e^r\quad fie\quad d_s\quad I\quad ,\quad JN\quad u^m\quad be\quad rT\quad h\quad e_o\quad y\quad 3^81(9_{91})35-_{-5}1\cdot I\quad G$

$aan\quad d\quad N\quad .$

$e_{dsI}\quad M\quad a\quad t^h.C\quad o\quad mp\quad .5^3(1989)689-69_6.$

$F\quad .\quad .Gant\quad m\quad a\quad h_{er,T}\quad h\quad T\quad h_{eory}\quad o\quad M_{\mathbb{Q}}\quad r\quad e_{I,C}\quad h\quad e_l^e\quad a\quad N\quad e\quad Y$

ork $N_Y.19\quad J\quad .\quad G$

$c_{re}^v\quad v\quad \mathbb{Q},\quad i:El\quad it\quad C_{ur_{es}}\quad n_d\quad R_e\quad ated\quad T\quad o_{ic}^p,H\quad .\quad K_{is}\quad l\quad evs\quad y\quad an\quad d^M.\quad Ra$

M u r $(\quad ty\quad ed.)\quad C_R\quad M\quad Pr\quad oc\quad d\quad n_{sa}n_d\quad L\quad c_{ture}\quad No\quad t^{es}\quad A\quad m\quad er\quad M$

$d_{ne}^c,RI\quad ,1\quad 4\quad 6_{--}8_3.$

D \quad R \quad .\quad r_{ay}\quad s\quad n_{,Th}\quad a\quad r\quad t^{hmet}\quad i\quad geo\quad mer

$5-_{-}1_2$

[H S \quad A \quad H_i\quad n\quad dr\quad y\quad a\quad J \quad H. \quad S\quad v\quad e\quad m\quad a\quad n\quad T^h\quad e\quad c\quad a_no_{ic}\quad a\quad h_{eg_t}\quad a\quad n\quad di\quad n

$e\quad g\quad al\quad p\quad oi_{ts}^n\quad o\quad eli\quad tcc\quad u\quad es\quad I\quad nv^e\quad n^tM_{ath.9}3(9_88)_419_{45}^0.$

S. L \quad a\quad n\quad g, D \quad iph\quad t\quad ne\quad a\quad p_r^p\quad o^x\quad ma\quad t\quad o_n\quad o_{nto}r_{us_{es}}\quad A_{mer}\quad J\quad M\quad h_{.8}\quad (6194

$521\quad 53$

[L \quad -\quad E_{lpt^c}\quad F\quad c\quad t_o\quad ns\quad ,A_{ddi_{sn}}^o-W\quad el\quad e_{yR}^e\quad ea\quad d\quad g\quad 1\quad 9\quad 3

$-_{-}-,El_i$

$e'\quad B_{ri},197-_{-},^C\quad onject_{ue_d}\quad di\quad o_{ph_{nt}i_n}\quad e\quad t^i\quad ma\quad e_s\quad o\quad n\quad eli_p\quad B_{rh}\quad e\quad '_{r}\quad B_{sel}193,1-$

$-551\quad 71.$

$A.K.L$

$r\quad t\quad o^na_{lc}o_{efficin}^e\quad ts\quad M_{\mathbb{Q}.A}^h\quad n.2_{61}1_{98}2_{,5}^)\quad 5-53$

[M \quad Y\quad u\quad I\quad .\quad a\quad M\quad n_{in,C^yc_{to}^om_{ic}fiels_{and}\quad mo\quad d_{lar}cu_rves,R_{us}\quad i\quad n_{\mathbb{Q}}^M\quad h\quad S\quad r_{v^{es}}^y2

(\quad)(19_1^7\quad ,7-7

[M \quad B\quad M\quad za\quad u_r,R\quad t\quad o\quad a_{lpont}^i\quad o\quad nmo_{du}\quad ar\quad c\quad u_{v^e}^r\quad s,in:Mo\quad ul^a\quad r_Fu^n\quad i_{ns}

o\quad O\quad Vr^{ia}\quad be\quad V_L\quad e\quad c\quad u\quad r\quad N\quad t\quad es_i\quad n\quad M

[P_{de} Weege *r-comma* Pro
_{iz} $R_a m_{an} a_{n--Na} g_{el} eq u^{a_{to}}_{in}$, M t C o $m^p 4(1986)_7 1_3--7_2$.
 $33 W_{59c.m} d^g_{tIn^{te}} eri_{nt} s$ on u v e s f $ge_n u s 1$

— — ..

C . L S *i*egel, Ub *r*^{eini}g An w n d
i J . H . S Al ver m a , Aq a n aiveve sio nof Si gelst eorem J Reine An
gew i n u t i_{t^t} r e ' h , .
M a $(h.37819^8 1,60 - 10^0$.

[S i cu r e s , dM a t h . C $c_{om^b p} 5ew_{5(1} 90,72 i 3 - 43.^t$ and e a n $o^n i_a^c h_{ei} g$ to n e l i p
_v
_{a n}
 $[^S M I A T H.^M u a l S^a a b_{ucke n_1} 9^{93} 2 3$
R . P $S_{t_{ei}} e_O n^M o r e_l s e$
M h . C o mp 46 $(8^6), 7^{03} - -7_{14}$.
R . J St r^{oe} k e a n d N

e t m a t n l i n e a f o r m s i e l p i c l $o^a_{gr} i_h m s$, A t A i t h $6(\frac{9}{1} 9_4, 177 -$
 $-19^6 NT a_n a s a d B M.^M d e W e g T d W - , a_{---, H} o w t o m e$
x p i c t l y s $^{eo}_{i_i} o l v 3_e a 1 T h u - -^M h e e q^u n$, C o m o i t o M_{ath} .
 $(19 2,_{22} 28$.

$[_d W t u m M_{oo.r} W i s u n d e n n f o m a m_{ica}, A i^p_{mste} d t a m^n e ,^e 187^t i$
 $.^{ns}, Ph. D T h_{si}, C_e r v k e I r t r 9 [Z D_Z a i e^r, L a r e$
n r p o n $o^{ne} i_p c c_u r^{ves}$, M t h . C o p $4_8(9_{87} 4 H .5 \overline{\overline{Z}}_{a3_s} .e$
haus , O n H e n s l f c t o r z a i o n , I , [Z s n e a i t J . N $u^m b$
 $r_{The^o} y 1(169) 29 311$.

[i h i g , M a . Z .147(1976ff), 5 - 51.b $t_w e_n t^h_{eW}$ e l h e i g a d h N
é r n _ - T a e h t $t^h 3$

[Z i — $O^n M a_{nn' s} c o n_d i i o n^a a_g o i h m$, $\frac{B}{l S_{oc}} M_{at h} F_{anc} M m^{.49} -$
 $(1977$

[Z i — G e n e a_{izt} n o f M a n i n ' s c o i n d t i o^n_{al} a o t m i_n: S Y
M S A C $7^6, Pr o . A C_M S$
[Z i a n d A m p l a t i o s , R . A . t M s o l i t n (d .) , K l u w e r 98^i 9, u 279 - 3 2 . n
 $N_u m^b_e r^T h_{eo, r} p i c n l e$
[Z i — , A $i^f_{mtom} r_{ula} f^o_r t h e c a n_o c n i a_l g_h o a_{ne} l_{ti} c u v a n i^s_a \pi_c$
t i B t e l i n , 1990, m_6

r 41 - -5.

F A C H E $C_{H9M} A T H E M A T I K L A B O R A T O R Y O_{RI} N_{FO} R M A T I C$
R S T T $D^{Es} S A A R_L A_N D_E S U_N I V E R S - I T D -660 A A R B R U C K E$
, G_E R M A N Y N A G Y E D $E^I K R T.9 H_{-402} D E B_R E - C N - E, H U$
 $N^G A_R$