

d H Z M R ru^c e^n
 S
 n o nan e^l lip t_c c u_{rv} e E o_{era} n a^l g e_{brac} n
 $i - n$ im $l - a$ $m - o_d$ e lof E o^{e-v} r \mathbb{K} , this n
 S $[Z_{i_4}, \cdot^s_{Thico}]$ c_t r e_{waspr} v e_d by S v e m a n [i 1 o r_e ipi c_{cu} v e w_{ith} i
 $n^{e^g}_{ra_i}$ mo du r i^n v_r^a ant j o r \mathbb{K} an b_y H i^n d rya n_{dS} i ve m
 $[H_S^i]_f$ r g e^b ra i_{cfu} n c o_n f_i ld s. O nh e o_t h e r a^n d b i n
 n n g w_{it}
 B a e_r [B e_{effe^ctv} e bo u ns f_o r t h s ze o t c_e^o ffici n_{so} f n_{tegal} p o_n
 on E h a v b_e en f_{un}^o db y_v ar o u_a uth o s (s [L 4]. he m os r e n t b
 o_{un}
 wa s e^t a b l hd by W . S_{chm} d t S^{ch} , T h.] H oweve r^t h e b o u n d s_r^a erathe
 l_a r e a n^d t ee o_{reca} n be^u s e o nlyfr ol v $g_{so_e^m}$ p a tc u r
 e quaio
 $(s^e e$ [T $W_{1!}, [tS]$ o rf o^r t e_a t_i n a p_e a l_m o d_{lof_e} i $c c^{u_{rv}e_s}, a^m$ e
 T h_{ue} c u e $s_o f_{eg}^d e_e$ $3s_e e_{[G S c_h]}$). T he S i_e el - a k_{erm} t h o $d_{(s} e e L^{3!} f_o$
 t h $c_{alul}tio$ o fi n e_g rp o_{int} o n e_{lip} i cc r^v e s $\sigma e \mathbb{K} = \mathbb{Q}$ req u r_{sso} m
 de ta i_e d i n_o r m_a i a $b_o^t c_e r^t$ a i qu r i n_u mber fie_d s Co m_p ut
 ng t_{he}
 fie^l s o $e_{nr} e_{pre}$ ent a har d pr $o_b e_m$ a n_d^* m orev $r^t h_i$ s a p p o^{ach} do
 n o eem_t o b^e a p pr p r a te. Th a i s w h^y i ge n_e r a l a ll^t h_r^e ultsm
 eno n_e
 a_{bo} vecann o b e us d f ort h^{ea} c t u $lc_a c_{la^t}^u i_o n_o$ f a l n t_g r l po n^{t_s} o n_a
 e_l ip t_{ic} c u_{ve}^r E o $v_{er\mathbb{Q}}$.
 H o w_e v_{ert} h_e^e s a not er meto d s_g^u ese d b y $La_{n_g}[1, [L^3_a] n d_{fur}$
 t her d e v l pe d by Z_a i $r_{[Z]}$. We s_a ll w_o kout the L a n - Za^{ger} m e_{ho}
 a d tu $n_i t_i$ n oa n a gl o_{r_i} hm f ro d t_e m n^{ing} a l i n t g_r al p o_{i_n} to n_{llp}^e
 t
 $r^{e^s th}$ c_{uv} s E ov e $r_{\mathbb{Q}}$ us $i - n g_e l$ i p_t cl o_{ga} r t h_{ms} . T_{he} l g o r i ht m e q u
 en ow

l e d_e^g of ab $a - s$ so f $h - t_e$

u_{ndfor} n e a r f or ms i n e_{ll} ti c l_o g rt hm s. C m $a_{red} t^o$ t e^{S_i} e el - B_{ake}
 T_h i w_{orkw_a} sp r $i_{a^ly}^{s_u} p_{pr} - t$ e $b_{yth}^D e^u$ t c e F $rosc_{h_{un}^g}^s e^{m^e in_s}$ ha , th e H_u
 gaia A caem y o S_{ce} c es a^{n^d} $t_{he^{S_i}} m_{nA}$ G

o \mathbb{Q} of r a k u
 t h e M r_d el_{-W} el^{group} $E(\mathbb{Q})$ A tu a_{ly} , thi i st^h e $onlyd$ s adv^{anta}_{age}
 ot h L ang - Z ge r_m e ho_d Ho e^{ver} a_n a g o r i_h m p o v i d i g s^{uc} a b
 s s w asr
 c_e n^t l ydv_e l ope d b y t h fi_{rst^a} nd thel a sa u_t or $[G^{Z_I}]$ t i b a s d o^{n^i} d_e
 o f M_a^n i [M] a_n d dep n ds o t h e v a l d t o f t h_e c o n e c
 $w - i_n$ r t o^n_{Dy} e ($s_{ee[}], Z 3], f_o r x a m p e) . W e_{re} p la a_n^n n g t o $m a_k$
 e i $i_n d_{pe}$ n de n_o^f t^{hi} c n e c u e T_h e s c o_d^n c o m p o $n_e t_s^i$ a_n e x p^i i l
 o_w b $b^{und} f^{or}$ ie a f m_{sin} el pt c $l_{os} a_{rih} m^s$ o a_{ge}^l bra c n u $mber$ s A ga i it
 w_{asony} $r^{ee} n_l$ y t_h t S D v i d D] e ta b $l_{sh} e^{ds}$ u h $a_{ne} x_p$ i it b o_{und}
 $t h_s$ p_r v^i g a o $h_{erco} n_j$ ct r e o L a $n_{.T}^g$ h s m_{eant}^a b_{rea} t h o^u g h n o
 e^n da $v_{or} c_{n^e}$ n i n $i^n t$ g^{ri^a} $o_{in} ts$ A n^{al} g o_s est $m^{a_{ies}^f}$ o l_i er f_{orm} i_{nc^o} m_{plx}
 a d^p adi $c^l o_{gar}$ hm_s ha db e n s $u^{cessu} l_y$ us ed f^{or} t_h e com pet e $e_{sol} l_u^u$ n $o_f T$
 $h eu$, T h u $e_M^- a_{hl} e_{rand}$ n d e x_{fo} m eq uao n_s ($e - e_{[}$ P_S Td W 1, [Td
 W 2, [G P P) T_e r e u^c t o $n_{pro} c_{ed}$ u e, b a ed on n^u $m_{er} cal$ d p_h a^{nin^e}
 a po x i m tio^n te c hn q u e, is ht_i^h ri m p o $t_{ant} c_o$ m po n^{ent} oou m_{ethod}
 Wesh all use h e^{rea} v arian $give_{nb}$ d e $W_{ege_r} [d_W]$
2 . H e ghs. T he liptic crv e E ver \mathbb{Q} s assu me d o be giv n i n
 sor $W i$ ersras i_{sno} rm alfor m u o i t e h
 $e \quad t \quad 2 \quad 3$$

$$(1) \quad E : y =_x +a x + b =: p(x) \quad (a, b \in \mathbb{Z}).$$

The $d_i c - r_i m n - i_a n t$ of E ov e \mathbb{Q} is

$$\Delta = 4 + 27 \neq 0$$

an d the $m dul a - r i v r - a i$

$$j = 12^{34} a 3.$$

By th e Morde $l - l -$ We $l - i$ Theo $r - e$ m ,

$o\mathbb{Z}$, w h e and edr

Le $t_{u-s} recal t - l_h$ e noto nof hei ht on $E(\mathbb{Q})$. Fo r a rat onal point

$$\xi, \zeta) = \gcd(\eta, \zeta) = 1$$

h igh t is $\eta \in \mathbb{Z}$, h_e n r t r e { l o m ax $\{\zeta, \xi | \}$ i f $P \neq O$,

$$h(P) = 02 g \quad | \quad if P = O ,$$

$$n \rightarrow \infty$$

null p c e of h issim pl ythe $to_{ri} - o$ n roup $E_{\text{tor}}(\mathbb{Q})$. T he $f - e_{ore,h}$ is a $posi^{t-i}$ v d $e - fi$ n i e qu ad_{a-r} ticfo

$$E(\mathbb{Q}) = E(\mathbb{Q})/E_{\text{tor}}(\mathbb{Q})$$

B y e m b d $i_n \cap E(\mathbb{Q})$ in th r_ d $m_{e_n} s^{ona_{rel}^a}$ s pa e $E(\mathbb{R}) \cong E(\mathbb{Q})_{\otimes \mathbb{Z}} \sim \mathbb{R}$,
 t_o

an c dthus g a v *hesxrie* n to a $o_E uclid^s$ ea e_{nnom} t o n $E(d_{\mathbb{R}}.i_{Int}rhem_E)$ u nci d
 $a^{\mathbb{R}}$ ns = pac i s r

nkenf mo geometry o fnu

L $e_t P_{1..}, P^r \in E(\mathbb{Q})$ de n ote s^u ch a b a i (oft e infi n_{i_t} p a - r t o f $E(\mathbb{Q})$
T h_e n e a_c h r - a io n_{al} p i_n t $P^{\infty} \in E(\mathbb{Q})$) $a - h_s$ a n^{iqu} e $r_p^e r_{sen}$ a o n ofh e o^m

$$(2) \quad P = iP_i + Pr + 1 \quad (ni \in \mathbb{Z}),$$

$$= 1$$

$$h_{er-e}P_r + 1 \in E(r - o(\mathbb{Q}))$$

We w ant toge trid $o - fP + 1$ in (2) T o th s^{-i} end , we m u ltp y
bot hsid o

$$(2) \quad P = \sum_{i=1}^{P-n} niP$$

N o t t_h a $t - commab$ y a_{fa} m o s h e r - oe^m o fM a $z^{u_r}[Mz]$, we h ve

(3)

$$b^O \quad u^w, {}^e a - E - nc_{ovep}^r \quad m_{has}^u n^e oto^n \quad r \text{ on ,}$$

$$\quad f$$

I n o e_r t c_o mpu e_{al} in g r a_poin s

$E(\mathbb{Q})$

(2) of mu P fiby the $b^n uass$ po nts $P^d(i o = 1, \dots)$. P u t t n_i in t_h e r e p r_s e ta to

$$i \quad i \quad i \quad . \quad r$$

$$) \quad N^i = 1 \leq ai \leq x_r$$

$$r - e \text{ rp se n t a i n } (2) \quad u - d \text{ lo t r oi n ,}$$

$$P = \sum_{n=1}^r Pi,$$

$$i =$$

(5) $(P) \geq \lambda - 1N$
 o n on - t $r_{si} \circ n_p$ it s $P \in E(\mathbb{Q}_w)$ e $0 < \lambda - 1 \in \mathbb{R}$ h e s amle
 t g $e^n v_{lu}^a$ o $f_{th} h_e$ m a $r_{ix} associa$ d w $t_h h_a$ n d the g $i^{ven} b - as^{s-i} P_{1..,P}$ r_o f $E(\mathbb{Q})$
 Nex t w e a - r e o - $g_i n g t_o r_e$ p l a c
 o o nd on $c_E(\mathbb{Q})$. T
 p ont $P = (\xi slash - zeta \ 2, \eta /) \in E(\mathbb{Q})$ s - i defi n d $a_s parenleft - c.$ Z 1] - Z three - i], [Zi])
 $\{1 m x \{\infty + 2 \lg \zeta, 1 g | \xi\} \text{ if } P \neq O,$

$$dP) := \begin{cases} 1\mu & \text{if } P = O, \\ 2 & \infty \end{cases}$$

w it hth e “ h e g ht ” of E

(6) $\mu \infty := \logmax\{||12, |b|^{1/3}\}$
 o rth $e^{th} ff$ eofficien - ts, $\in \mathbb{Z}$ of the ellp ccuv - eE.
 $r_{een} tal - bh^s d$
 $(c \ a_s \ [Z_i]) [S2] :$
 $(o \leq d_{(P)} - \hat{h}P) \leq 2^3 \mu^\infty +$
 I n f_{act} o n c_m bi n_{nt}^g h_e h e g t e s_i ma t e^s o_t^b ained i [Z₂] and [Z
 $w_{th}^i t_h$ s e r m Z 5 , one n d u pw t h^t h sig tl y st_o^r n_g e re t^m a
 e

$$(7) = \frac{7}{1}$$

F i m pli i y - comma howe e , e - w $h_s ll_a$ u s (7) ra h_e
 $h_{at} h_e$ ei ht di^n [Z i - - z 3^d] i ff rs f_o^r m_{th} e d_d e fi n d_a b^o v
 e by a $f^{a_c^c}$ o
 $\overset{\text{o f .}}{e} \overset{\wedge}{2}$

) - 3 2
 H e n $c_e f_o^r$ $s_u f f i_i e t^{l^y}$ $l^a r^{g^e}$ i $n^t e_{gra}$ p o i t s_P = (ξ, η) $\in E(\mathbb{Q})$, m o_{rep} re
 c i e

$$)$$

$$\text{e lds}$$

$$(nine - parenright 12og | \xi | \geq \lambda 1N 2 - 2l_g 2.$$

W er ema rk h a f μ slar g - e e g ex p (μ) 3 > zero - one 6 nd if $0 \leq w \leq m$
 ate s ()₇ - - (9) a s o_l low s.

$$a_{lp} o^i ns \quad P \in E(\mathbb{Q}) \quad i_{nt} e_r^g$$

$$\text{ow er es im a t e } hP) \geq h(P) - \text{e nc}$$

$$(8i) \quad 2\log^{|\xi|} \geq$$

$$\text{T h n t h e a e } o_{f0} \leq 1 \text{ o } g || \leq \mu \infty(9)_i \text{ t o b e r}$$

$$(9i) \quad 1 \log |\xi| \geq \lambda N - 2 - 2 \lg 2 - 1 \mu .$$

$$\text{T h n ex r a } e_s \text{ r c .}$$

$$\begin{aligned} & \text{W ec}_o \quad n_{fi} \text{ eo} \quad \text{urel e s o} \quad e - x \quad l_a \text{ i n i n} \quad t_g \quad \text{h s}_e \quad \text{rc h} \quad p_{re} c_e u - d_r \text{ ef} \\ & o^{rlarg} i_{negr} \end{aligned}$$

$$poi_{nt} \quad P = (\xi, \eta$$

$$\begin{aligned} & e - l_{li} \text{ ticl og} a^{i-r} \text{ th m of } P \quad \text{by} \quad u s - i \text{ n } gt_h \\ & \text{cu rve} \quad E(s - e \text{ e [L 2] , f or exa mple }) . \text{ The re ex } s - it - s \text{ a } a - l \text{ ttic} \\ & \Omega \subseteq \mathbb{C} \text{ such tha} \end{aligned}$$

$$\begin{aligned} & \text{the g o } p_u \text{ of om p } e - x \text{ po nts } s - i \\ & w_h \text{ e } r_e \quad \Omega = \langle \omega_1 \omega_2 \rangle s_g \text{ n } e_{ra} \text{ ed} \quad b - yt_{wo} f_{un} \text{ d amental pe r } i_d^o \quad s \omega 1 \text{ an} \\ & \text{d } \omega 2^o \end{aligned}$$

$$\text{w hich } \omega 1 \text{ s re l a n d } \omega 2 \text{ c om } p^e x . \text{ We p t } \tau = 2/\omega 1 \text{ an d s u me w th_o u}$$

$$\begin{aligned} & l_o \text{ s of g e n rat } y \text{ h } a^t \text{ I m } \quad \tau(\\ & s^p \text{ e o } \Omega \end{aligned}$$

$$P = (\wp_{\text{parenleft}} - u \text{comma} - \wp'_{\text{parenright}}) \leftarrow u \mod \Omega$$

$$\begin{aligned} & \text{s oth at h e c oo din es of an int ra po t } P = (\xi, \eta) \in E(\mathbb{Q}) \text{ a r givenb t r} \\ & \text{a t e l i n } \eta \text{ e} \end{aligned}$$

$$\xi = \wp(u), \quad \eta = \wp'(u).$$

$$\text{o yn o mial } p - \text{parenleft}^t x \text{ i n one - parenleft} \text{parenright} - \text{period} \text{ The } t_h \text{ (c .[Z } \\ \text{a bracketright} - \text{parenright}$$

$$\begin{aligned} \omega = 2 \int & \sqrt{x} \\ \alpha x + a_x + b \end{aligned}$$

$$\text{The eli p t i c lo gar hm o f } P = \text{parenleft} - xi, \eta \in E(\mathbb{Q}) \text{ is (f . [Za) 1}$$

$$(11 \quad \omega_1 \xi \quad \sqrt{xplus} - a - x - plus - b3$$

$$\mathbf{braceleftbt} \quad \sqrt[3]{2} \quad -1$$

$$\text{ad} \quad \text{s a} \quad \text{e a} \quad \{2m_{\alpha+} \quad M \text{ In} \quad \text{t e l } alpha - a - p - r - comma \quad beta - parenleft \quad + \quad M \quad i - f \\ ic \quad logar - e - s_u.t \quad mofh - te$$

$$(\xi,\quad \mathbb{Q})s_{a_t}s \quad \begin{matrix} \textit{eset} & \textit{teothi} \\ \textit{fi} & \xi>\max\{\,,\,\}\end{matrix}$$

$$T\,he \qquad n \qquad 0$$

$$\int\limits_{}^{\infty}$$

$$4) \quad \sqrt{3-d} < \sqrt{\xi}. \\ \text{n } i-t \text{ must} \quad \text{b e } b_o$$

$$o_{th}$$

$$\leq 12 \quad a_n \text{ da s} \quad u-m \quad n-i \quad g \, \xi > \quad \begin{matrix} m \\ m \times \{0,0\} \end{matrix} \quad \text{weobta nf o} \quad m \quad (11) \\) \text{ an d } (14) \text{ th}$$

$$\text{estimate}$$

$$5 \quad |u_<-\omega 1 \cdot| \, |\xi| \, .$$

$$\text{O n} \quad \text{cm} \quad \text{bini g} \quad (9) \text{ a nd} \quad 15 \text{), w e et} \\ \text{o n} \quad (\sqrt{g}$$

$$\text{E x } 6 \quad \nu 1_e \quad - \lambda 1 \text{ fo r}$$

$$c:=2-2\cdot\sqrt[4]{4}.$$

$$\text{N o} \quad \text{y} \quad \text{t h } c^r \text{ uc i a} \quad T_h e_m^e$$

$$= 1n_i i + ur1+$$

$$\sum_{i=1}^r$$

for som e n_i e $e_r n \in \mathbb{Z}$. If we apl ce (2) b y (2), weo bt an ort h $e - l$ lipt l g rthm $u_r = g u_o - f$ th ep i - o nt $P' = g P \in E(\mathbb{Q})$ the reprsen

$$(18) = \frac{'}{0} + n - prime - ii \quad (n = g^n)$$

w h i - c h weshall u_e i s_t ad of (18). O f c o r - u s , (16) $i - s$ then t obe e - r p la ced b

$$\begin{aligned} & \text{parenleft - one6t)} \quad | u \tau | < \text{gcexp} - \lambda 1N2 \} \\ & H^e \text{ r } u^m \text{ e } te_h \text{ e } i - p \text{ i } l_g^o \text{ t } h^m \text{ s } n^r \text{ m } i - l \text{ z} \\ & | u_i | \end{aligned}$$

S n c eD vid w ork sw i tht he c l as s al W ee r s r a s s fo r m

$$\begin{aligned} E : y^2 &= 4x^3 - gx - \\ & \text{w e must r ar a neit tog e t} \end{aligned}$$

$$\begin{aligned} E : (212) &= 3 - 1_4 g 2_x - \\ & \text{s ot at we ave } g = -4a, g = -4. \text{ Hence , th eh g} \\ &) \\ & = h(1, -4a, -4^{b-comma} j \\ & x \{ 4 | ap, | b |, | p \} + 1 \text{ g m ax} \end{aligned}$$

$$\begin{aligned} & p \\ & e \frac{t}{h} es^u \text{ m m a t o n} \\ & \text{n } \mathbb{Q}.^z \text{ W itin gh } l_e ac m \\ & \mathbb{Z} a - n \text{ dus } n - i \text{ gth e s um f o rm u l a} \end{aligned}$$

w eobta nfor h t hee x press o - n i i

$$\begin{aligned} (02) \quad h &= h(\sum -4, -4b, 1j_2) \\ & x - braceleft 0_{lo} g | 4_a p_{,lo} g | 4 |^p 1 - j^1 p - o | j_2 \\ & + m a_x 0, l^{log|4^a|}, \end{aligned}$$

$$2| \quad bar - j_1 \parallel j \; | \},$$

$$\text{eh} \quad \text{x } p - one - r - eion^a 2t_0) a \frac{r}{s} s \text{ t h e val u e } 2ah \text{ i } r_n \quad \text{D av d} - i \text{ v s y T h e o e}$$

$$e_{mT} 2 \frac{h}{.1} . \quad \text{F } c - e \text{ with (2) := a n d } 1_{(2)}, \quad \text{e a n } m^{be} \text{ s } V1.. \quad V$$

$$\{ \quad 3\pi \mid$$

$$(21 \quad ol \text{ g} \quad Vi \geq \quad m \text{ a}^x \quad h(i,)h \quad \omega \text{ Im } (\tau) \quad f_o 1 \leq \quad i \quad \leq r \text{ a n , \quad a or } t - i \text{ ori} \\ , \quad 1$$

$$(2 \quad 2) \quad B$$

$$\text{n o byt } he^b n_e \text{ nii } a - o^y \text{ n } (4o)i_o \text{ f } o_{seanot}^{N,weha} e - v \quad \text{rth e } o_o \text{ e offi i t } h_e \text{ e } t_i \text{ m a} \\ \text{ets}$$

$$(4I \quad n - i \leq gN \quad for 1 \leq i$$

$$\text{i nerti ngn } h(6t \text{ th , eexp i r ss } eon(18t \text{ for } 8ut \text{ w ege } e_t^e s_{ti} \text{ ma e } a_f \text{ o } l^o \text{ ws O i} \\ 1 \quad) \quad \text{\bf vextendsingle} \quad \text{e r i} \quad \text{\bf vextendsingle}$$

$$iequal - one \quad i \quad 1$$

$$a_ndt_h \text{ e r - i g } t_{han}d \quad i - s_d \text{ e } c_a \text{ n b } e^m \quad a_d \text{ e} \leq 1/2 \text{ f } r_s \text{ uffi c in } t_l \text{ y ar e } N, n_a m_e$$

$$for \quad \checkmark \\ N \geq \quad g\lambda 1 \quad c.$$

$$H - e \text{ n } c_e \quad w \quad e_o b_{ta} \text{ n}$$

$$= i - r - i - vextendsingle - vextendsingle^{n0} \\ \leq \quad 2 +$$

$$\text{a nd}^n \text{ o } N^r \text{ m} > ieew - o - e - nchoo^1 \quad) \text{ f } h - t \quad w_s \quad \text{n d} \quad b \\ (parenright - comma(2) \cdot \quad \text{h e } f - e - rr^e, \text{ a i t h } n^B :=) \quad 2gN,o$$

m a $t^e(1^{6'})$ w it h D a_i^v d ' s T v in P R op_o SI IO N . T he e $l - l_{p-i}$
 tc logar t hm

$$\sum_{n_0} r$$

$$i = i - r +$$

of an inegr a l p o int $P = (\eta) = (\wp(u) \quad \wp'(u)) \in E(\mathbb{Q})$ such that

$$\xi > \max_e e$$

$$0 \frac{a}{s} i)(three - parenrightbigg - parenright(^t i_s fit_{(h^e}$$

$$\exp(-C^h + \log 2 gN + 1 \log g \log 2 gN + 1$$

$$\leq |$$

wh re $N = \max \leq \{|i|\} a s i n 4$, thec ons a ns $\lambda 1$
 $(\text{ki } ({}^1 g - \text{seven ar } s - h \frac{\text{etv}}{s} \text{ dom i t i - n g th em } u - d^j c_e \text{ t rm three - l o}$
 $|g e - w \text{ co cl u - d C o lar}_{(y} U_{(} n e r t \frac{y}{eh})p t^h e) (s_o \text{ the } p_{(r o p s_i o})$
 $),)$

$$\begin{matrix} r+1 & r+1 & r & +1 \\ 2 & 2 & & \end{matrix}$$

$$= 1 - i$$

Ab o u-a nd $e - f - \text{tr} - \mathbf{i} - \mathbf{n}_o^t \mathbf{e} - \mathbf{t}^g \mathbf{r} - \mathbf{s} \mathbf{l} \mathbf{p} i - \mathbf{o} \mathbf{n} - \mathbf{t} \mathbf{t} \mathbf{s}$. Of o-period $- cu - r - s - e^e t^{e-h}$ o n - e q r - u - e

$$N$$

h e - ffi - pc_{upos}^s, we t fir^e s^r s tea^t o
 $s L e - m MA$ period - two L t ϱ δ and σ erealn mb e - r s sat_i
fyn g

$$2 \quad \delta$$

$\varrho \geq 1$ $\delta \geq 1$ a n d $\sigma >_m^a x$ $\{(e/\delta), 1\} T$ en the $l - a$ r et sol
 $i - t$ n $x - zero \in \mathbb{R}$ of he e u - q a - t ion $x = \varrho + \sigma \log \delta$ x satsfie

$$\begin{matrix} & \text{thei - n} \\ x0_< & 22\varrho\sigma lg(\sigma\delta) \end{matrix}$$

$$\begin{aligned}
& 8) \quad \text{og} \quad r + \quad 1gN \quad + 1 \quad \text{og og} \quad r + 1gN \quad + \quad 1 \quad r \\
& \quad \quad \quad (\quad l \\
& \quad \quad \quad < \quad \lg r^{+2} \\
& \text{O } bs_{e-r} \text{ v in g (1 7) an d } \quad \text{two - parenleft five - parenright}, \text{ w e} \\
& (29 \quad c - one := m a x \{ \lambda one - g), 1, \quad c^2 := m_a^x \{ \lambda_1, 0 \} (h) \\
& \quad \quad \quad \text{m b ythe } r - i \text{ th ha nd s ideof two - parenleft8)} \\
& \quad \quad \quad d_{erivef_r} \quad m (2 6) \text{ t hei } n_e \text{ qu } l^{ty} \\
& N \quad o \quad \text{ww e a ppy Lem } \frac{to}{a} N(\quad . \quad \text{L e } N0 \in r\mathbb{R}^N \text{ b e t } \quad \text{e agr } e \quad \frac{l - uto}{o} \text{ of} \\
& e_N \quad > N^i 0. \text{ Ta } akn \quad g^{ed} \text{ b } \quad y e \quad t ing \quad o \quad d \quad o \quad 0 \quad .T \text{ h } \quad (\quad) \quad n o^{th} \text{ o} \\
& \quad \quad \quad i
\end{aligned}$$

$$c^1 = 2 \quad n^d \delta$$

$$a_{Lm} \text{ m } \quad \text{a 2 for } N_0 \text{ te } \quad \text{s i m a te t } \quad \text{e s o } \quad \text{L m m } \quad \text{2 i } \quad \text{s } \quad \text{e } \quad \text{w } \quad \text{e } \quad \text{r f r}$$

$$\begin{aligned}
& h \quad r2\sqrt{r+2)/2} \quad r+2 \\
& N
\end{aligned} \tag{1}$$

$$\begin{aligned}
& \text{p } o_{s_i} t_v \quad n \quad \frac{e_g}{e} \quad r_s \quad \text{s t } \quad y \quad n_g (3 \text{ a } 27) e_{onN} \quad e_{,w} \text{ easo hav e } \quad N \leq \quad N \quad 0. \text{ f } \quad , \quad \text{b} \\
& h^e \text{ c n } \quad n \quad s \quad) \\
& 1 (32 \quad N_1 > \quad \text{m x } \quad \{ e^e, (6r + \quad 62, \quad 1 o_g (2c1) .
\end{aligned}$$

$$\lambda 1$$

$$\begin{aligned}
& \text{O nc om b } i - n \quad \text{n g } \quad \text{t erelati n } (_3, (_2 \text{ parenright - two }, (24) a_n \text{ d (1) } , \text{ we } \quad \text{th} \\
& \text{usar i ve } \quad \text{tth fol ow } n - g \quad \text{n } \quad \text{d } a_m \quad n_{al}
\end{aligned}$$

$$\begin{aligned}
P &= r \sum_i n P_i + P \quad \text{relement} - E - Q - \text{parenright} \\
i &=
\end{aligned}$$

$$be \quad a^n \quad i \quad t_{eg} a_{lpo} \quad n \quad o n \quad t_{he} \quad el \quad i - n_{t, Thent} \quad e \quad m$$

$$N = ma - x_{r\{\}} bar - i \}$$

t o (21 .

a $y_n e - l_{i-l} p c_i$ cur $v - o - v - e \xrightarrow{er} E \otimes_{\mathbb{Q}[e_{lo}]} r - a$ o
 $s - i_x$ o er \mathbb{Q} . Howeve , a n y i m p o - r v m en ofDa vi ' sb o n di
n [D] w o ld mak

$I - t$ re ma ns t o $prove_t e - h$ twol emm ata , t oe xplain ho wto c lc
 $u - lt - a^e$ th

e lip $t - i$ c lo arithm s ui of h e b as spoin s Pi as w ll $s - a$ t h er al $a - n$ d
 $co_ml - pe$ p e - r i od ω_1 n d $\omega_{2,r}$ esp ct $v - i$ $e - l$ y , $o - st - h$ at t e Vi s c
n be d t m n d - e i na c o

d nce $w - i_t$ h two - parenleft1), an dto s $h - o$ wh owthe bo undin t $e - h$
The o r - e mc n - a ber e - d uce ofac $i - l$ it t e the c o mputa ion of a $l - l$
inte gralpo $i - n$ tsin

5 . P r o o fs

P ro f o f L e m m a 1 . We may ssu me w ih u t os sofgene rl
t yh a

he l rges t r e a loot $\alpha \in \mathbb{R}$ o fhe pol no mi l $p(x)$ n (1) sn onn e g aive .
Fo

i α i a_{sne} g tve , wetranslat e p b y a y sutab lepo i tv enum

$$\leq \sqrt{32 - 1} \leq 32 - 1,$$

sn ce , by 6),

$$i \left(\{ \sqrt{ \sqrt{ \{ \{ \sqrt{ \sqrt{ \{ } } } } } } \right)$$

$$|p - bar| := \max |bar - a| / 3, 3_{|b|} \leq m ax |a|$$

$$(\alpha) := p_{(y-)}^{\geq M - 0C} M_h i_n^{o-s^e 0 \in y}$$

an d $u_p p^o$ s

$$xi - greaterm x 0, \xi 0.$$

$$O_{ur} \in n_t g^{ra} l^{bec^o} m s$$

$$\int x_d \int \infty dy$$

\sqrt{p}

$$z := y - (\alpha + M) \quad in$$

$$(q + (+ = z +$$

$$>_0^3 ud_{greater-zero}^{nd} \quad e un \quad ec_{o-e-s}^{rt} n \quad . \quad U on - c \quad so_{i-t-i-d-n-c-h-t} b \quad ndo \quad o n w$$

$$4) \quad \sqrt{() < z32 = \sqrt{\xi_-} \alpha.}$$

$$\xi \alpha \quad r \quad z \quad \xi_- \alpha$$

N o i $\beta\gamma \in \mathbb{R} - \infty \gg \partial$ we de ve rom $\xi/2 > \alpha + M/2$ y (13) and (3) th t

$$ich \quad -\frac{l-ds}{\sqrt{\xi}} \quad t \alpha \quad \sqrt{sse} \quad +\frac{M}{o} \quad em^\xi \quad 1$$

$$\beta + \gamma = \beta + \beta < 2\alpha, \text{th e same con u i n h - o ld s}_i \text{ n c ea ai n } \xi/2 >_{\alpha+}$$

$$\in_{\gamma}^{z \geq} + (+_{nd}^{beta-one-parenright+} / \gamma_a \quad df = \beta$$

$$(z +)_1 \quad z + \beta_1) = z2 + \beta(1 + \beta_1)z + \beta_1\beta_1$$

$$\beta \quad ()2 \quad ()2 \quad ()2 \quad ()2$$

A l g e e , r o r $z > 0 (\Leftrightarrow x > \alpha), t - hs - i \leq 1$ ea sto the ine $q - u$ ali t
 $n + \beta_r)(+ a > a + o 2 :$

$$\int_{-\infty}^{\infty} dz \quad \text{integraltext-infinity} \quad dz$$

$$z) \quad \xi - \alpha \quad (+ \quad (\beta + \beta_1) \quad parenright - three \quad = \quad \xi - \alpha \quad +$$

$$B_u \quad t \quad i_{nt} \quad h \quad s \quad c^{as} \quad e \quad si \quad c \quad e \quad by \quad (13a_n \quad d \quad (3),$$

$$> \xi \quad 22_{M_2}^2$$

$$-\alpha \quad (1 \quad + \beta \quad 1 \quad 2 \\ \text{P r oof}$$

$$\delta_{=}^{x-zero-delta-parenleft-rho1-one-slash^{<2}+\sigma} 1\delta_{fi-es}^{delta-sigma-parenleft-g-o^\delta}\delta)$$

S i_nc_e_g an d σa_e^r a t e^{a_st₁} w_e hav e a n t isimp lest ρ_{he}_a+_{s-s}r_σ^eδ_{ei}^og_{nequa}_{t-i}^{σδ}≤_i2_{y.}ρ δ^{1/δl} g σδ),
n_o S c t i on 3 w e ne edto de t e m i_n a c or d n ewi t h t he c o dt o_{ns}(
ω, s r s pect ve l - y, th^t u g i in^E g τ =) ^an_{ω/} o ω1·h_T ω2, w e choose or o ur elipt c
cu^{r-v} e the_e

$$v_2 = r() = z(z + \beta_1(z + \gamma_1))$$

a nd applyth em e h o dofarith m t i c - g o m e ric mean of Ga us a sde sc ibe by Gr ayson Gr . F r h e c - o mp u at o ofh eel pt c o gari hms uif - o th p oin s P (1 ≤ i ≤ r) we u s et hefa tⁱ - nonv e gi n s - e ri e l_g venby Z a ge [Z f o mu a (10) Of ou se , t he Neron T s_a eheig htft heb

$$d \quad n \quad (21) \quad \sum_i^s \quad c a c - u a - l \quad t e d \quad b y \quad t h$$

6 R e^d u tc i n o ft hei n_ta₁ b ou nd . T h u p^p e b_{oundf} r N^o b t^a i n int h_e T h e rm isin g e_n r lto^ol_{arg}_f o_r^{c_o} m p u_tn_{gal} i^teg_r a_p s o u e l p i_c u r^v E o ve r^Q. H_o we v e , b y n_u m e_i a d_io_{phan} n^{ea} p p_{rox} m a t_o nte c hniq^{ue}

$$e \quad u_l t_y \\ \text{vextendsingle} / \sum \quad \text{vextendsingle} \text{vextendsingle} \text{vextendsingle} \quad / \quad 2 \\ \text{vextendsingle} \text{vextendsingle} \text{vextendsingle} \text{len}0 + \nu i \text{vextendsingle} \text{vextendsingle} \text{vextendsingle} < g_c \exp -^\lambda 1N \} \quad (35) \\ i = 1 \quad / \quad / \quad /$$

o btai n_e di_nh^e P ro_po_s ti o_nf o r_u = g_b^u y vⁱ r_u^t e of(1 6_a) n d (18)^{,t_o} e t - h_e w t h t h e es t Th o e - m_e m m a y e - b reg arde dasa h of expo ne n t - i_{alld}_{o-i} p ha ntn e e u a_to - n^s, a nd me ho d s f o rso v i n g_t h e m hav

$E(\mathbb{Q})_i s^t$ o $i_o - r e_{or^{if}} h^{eu}$ p p boud $N2$ for the c_{offi} $c^{e_i^n}$ s^o ft eba s
 poi n s i_{slag_e} .

$I_{nh_e} s_e q_u - e_l w^e$ ar e ogi n g to g_i e_{anoutl} $n_e f_e$ W_e e r_{sm} eh od
 $[d_W ap^p li$ $d_t o$ te p e $se^{ns} i^{t_u}$ a_{to_n} $L^e t C$ 0_{beas} utabl p^o $s^{t-i_{ve}} i_{nt}$ g er $^a n_{\Gamma}^d$
 b e th la $t - t_{ie}^c$ spa n edb y t h er $_o$ w s ot he $(r + parenright - one \times (r + 1))$ m a^t rx

parenlefttp 1 0 ... 0 **parenrighttp**
parenleftbt)parenleftex 0 ... 0 1 **parenrightexparenrightbt**
 C $0u1]$... C $0_u - 1] \lfloor$ $C0ur]$ $C0$
 r

7 d $(^w\Gamma \geq u_r d$ 5 + $f_4 N$ a $p - o(3.22)$) c^a n n o h f n
 w h n e a e e a 3

$$\lambda 1 \frac{\sqrt{\sqrt{\sqrt{N}}}}{\tilde{\omega} N} < \leq N$$

T o fi nda si able \tilde{N} , on $h - c$ oses C $i - n$ heor e r o mag n
 tudeo $N_2 r_+$
 a nd c len $h - o - a - minus - v - r - e - slash - l - two - e - parenleft - t -$
 $parenright - a - k - e$ N w w $\Gamma \geq^2$

N
 I f $\tilde{wide}N \geq N$, w eo $_b$ tan th e_{est} i m_t e
 $\sqrt{\sqrt{\sqrt{N}}} \leq 1$
 $\lambda^{1\omega}$

T
 i $s - ie_r t - a_e$ du tilthe u p er bound
 r a - e - v - s - e - t - t - i - a - t - l - e - r - l - s - r - p - a - parenleft - t - n - i - o - zero - n - n - commas ctoio t

$parenleft - two) f_{orie_gr} i - ly.$

7 . E x a m p l e s EX A MPE 1y2 = x3 - 1 620three - twox + 62877920 L 4
4

$$\begin{array}{ccccccc} \text{We} & \text{a} & k_e \\ r - a_{sso-fm} & \text{e} & \text{a} \end{array}$$

$$\begin{array}{ccccccccc} E & : & y2 = x3 - 1642032x + \\ \Delta & = 11two - five7102923^77_{94}28_35^2 & =^2 & 2^3 & & & & & \cdot 511_445_075_7, \end{array}$$

mo d ula $r - i$

$$= j^1 = 224933_{one-nine}718496$$
$$\begin{array}{ccccccc} \text{h} & \text{o f } E & 1/ & 2 & /3 \end{array}$$

$$\begin{array}{ccccccc} \mu \\ \text{odebwo}_l^{Ep}(\mathbb{Q}) = \{O\}, \text{ so that } M \text{ is a basis of } th \\ \text{W} \quad \text{a_l_s o} \quad \text{dis p a y} \end{array}$$

$$\begin{array}{ccccccc} P1 = (432108), \widehat{h}(P_1) = _3.363710642_5 \\ P2 = _6^{(39)}6_{37}2, \widehat{h}(\text{parenleft}^P2) = 33_88408_52_9, \\ P3 = (360, 980), \widehat{h}(P) = \text{zone-four}293916two-zero, \end{array}$$

$$\begin{array}{ccccccc} P = (108 & 212 & \widehat{h}(P) & 3. & 559132453 \\ P = (362 & 3868 & \widehat{h}(P) & 3 & .595291970 \end{array}$$

tr c m atix o fth e n ear f r m a s ociatd w
i

$h - a_b w^n i - o f ac h T z a nak i$ p a^{p-e} s comma - op^e o r ed d o nanag a_o
a
 $e_l p^t$ i $cc_u r - v$ esby m s oe l i tⁱ l g - o arthms .Af_t
im w e h p $p^i n^t$ o a T n k .

$$\begin{aligned}
& 3_4 1_2 9 \quad 1 \\
& -12380^{16} \quad 8617 \quad zero-period_6 1460 \quad -.2_1 851 \quad -1576 \\
A - e - tv^{x_x}) a_l^A &= {}^h 6_a^x \quad he_{25_A - 2_0}^{c, 8501} har_{.are 2937} \quad + {}^{-90}_{t_0^{r-e} i_5^{s-t} \quad 2ic} \quad 1429 \frac{2386}{5^{y_4^{atom}} g_{a-i}} \quad + {}^{7x^4} \frac{226^{-61}}{5^{g^2}} .2 \quad 7_{638.}^{763540} x \\
& \lambda 1 = 0.432372401, \quad \lambda 4 = 4.
\end{aligned}$$

$$\lambda 2 = 1.564757846 \quad \lambda 5 = \quad 5 period-five 147069_9,$$

$$\lambda 3 \quad = 1.9944767 seven-nine, \quad \lambda 6 \quad = 7_{zero-period} zero-six 831_1 531,$$

$$\begin{aligned}
& \text{of which } \lambda 1 \quad i-s \quad \text{ne d d i} \\
& r_o \text{ d i v n} \quad \text{by (1 0 i }
\end{aligned}$$

$$\omega 1 = 1 \quad 8_2 6^7 843$$

$$\text{xp } r_e \text{ o dis tha t}$$

$$= \omega^2 = \quad 0.3843290$$

$$\begin{aligned}
& \text{He nce thecons an tin (1} \\
& \quad 1
\end{aligned}$$

$$1_\lambda \quad , \quad 1 \quad = \quad 1^4$$

$$I_n \quad (1)_w \text{ en} \quad e e_{dt} \text{ h} \quad ee \quad l^{ip} \text{ t} \quad 1 \quad ogath \text{ m}^s \quad o \text{ t e} \quad bass \quad p \text{ o i t } s_{P1, \dots, 6}^P;$$

$$4_{95} 8842^3,$$

$$u = 0.zero-zero-nine-six \quad , \quad u - u = 0_0 2047900793_2 \\ 0_3$$

$$\begin{aligned}
& \text{a nd} \quad \text{t eq uant tyf o m (20) ,} \\
\text{lo m a } x - braceleft \quad bar - fouraj2bar - comma4 \mid b_j \quad j_1 j, 2 \} &= \quad 4 period-six 3145_3
\end{aligned}$$

$$\begin{aligned}
& \hat{\pi}_i \quad 3\pi u_i
\end{aligned}$$

) 7 and (2^2). It turns out that all $C \sim 7 \cdot 10$ and t_{here}
 $\leq \frac{2}{h} + 1 \sim 2.5$

$$\lambda - 2 = 1 - 2 - \lambda$$

Final $l - y$ in (1) w.g. $e_t - N1 = 1.11012$
 $d - e$ Th o e - $m - s^h w - Ns - less equal t a - N t - m - a - h7\} =$

Now wea p p y de We

0 0

parenleftex 0 1

0 0 1 0 0

parenleftexparenleftex 0 0 0 0 1

$-n - nC0 = \frac{i-w_{e_{t-h}}^C}{2} 0^{one-two^{e_{i-l}^{v-a_C}}} \text{toe uretha}^{t^{C=N^b}} (C_l^{w-e} 36) \text{ is } f_i^C \text{ tsfied. } C_{1_f^c} \text{ reduce } C_{0_u^t} \text{ he seven - t - l - zero - e - t}$
 $t_h \text{ e sh erts - t v e}_c \text{ or } b_1 f_t e^h \text{ a ttic t } \| b_1 \geq 114 \text{ seven - period n}$
 $\text{dy ie } d_s t^h e_b o u_n$

$$N \leq 10$$

A th_i dre d $uc^{io} 1$ ed s o h es_a
 d_u t on one $c^{tors}_{ofr} ea$ $n_u m_b e - r_{sn} s e d - a$ o_{in}^f gr a l_ν m b $e_r s a_s i^n$
 $W e e - r' m e h o d$, h_e upper b $o_{un} d$ isi m_p ro ved t_o $N \leq 8_{.T} h s$
 $i_{mp_r} o^{vm} e n$
 $i_s a c hie_{ved} \sin c_e$, $y p a - s s n - i_g t ot_e^h r_e l c_a s$, $n eo b ai n - s a s$
 $ha_r p e_{rstim} t - a f o t h_e$
 $O ur t s k_s t_{ee}^h f_{oretot} e s - t ornt g ra^{it} y_{al} h - t e p o - i - n_t s$

$P = nP - 1 + n2P2 + n3P3 + nP4 + n_5 P5 + n6P6$
 $4 \leq |n_i| \leq N \leq 8(1 \leq i \leq 6)$. It the r_e ma is to te e^{st}
 $\text{the e po } i_n$

$P =_{(\xi, \eta) \in E(\mathbb{Q})} E(\mathbb{Q})_{such that}$
 $\xi \in \mathbb{Z} \text{ and } 0 \leq \log |\xi| \leq \mu \infty = 7.15722 \approx 52$

$$6 \sim \beta \leq \xi \leq \gamma$$

in a_{cc} or $a_n c_e$ w it t h e_R ema rk fo_l^o w n g_L e m_{ma} 1_{.B} y t hi s e x t s_e a r
 h o w vr w e d dn o_t fin d a y_n e w p^o n s
 A^l og eh e_r w_e o^{b^t ai_n} d the 7 0_{in}
 in t-e g r a l po n-i t s n $\frac{E}{(} Q$ parenright-period
 R e mark . T a-b le 1h-s owsth at theact u-a_{lbu-on} d o-r_N is 2 rat
 h rtha n 8

Table 1

	$N^o.$	P	$n1$	2
2	(-1431, 99)	0	0	1 0 1 1 5.701 _{nine-nine} 5 0

minus - minus - minus - minus - minus - minus - parenleft-one - one - one

8) 0 0 1 4 99 1 1

zero – five – five – eight – nine – nine – one – two – seven – five – two – two – three – three – four – f

o_{mM_e} sr $[^M]$ e o f^r a k $r = 5$ o e r \mathbb{Q} a nd

form ab a isof

$$N1 \cdot 1^3 1^0 \quad \text{and } C \cdot 0 = 0600 \text{ w e}$$

$$\begin{array}{c} tupo_p \\ n \quad d_t o \quad N \quad \leq \quad . \\ sr_am - e \end{array}$$

W e_its h e fir_sc_{oo}r_d in e s $\xi_h^{oft}(2\times)$ e_e r s_{ras}^ts^m o d l_o f E v e r \mathbb{Q} w hi h
c_o s it^{ute} e e t_o f_a i n_{t_{eg}} r l o_{int}

$$\begin{array}{c} -4_{88}, \quad -68, -4_3 2, -43, - \\ -3_{six-comma} \quad comma-four \quad three-six \quad two-seven, 8one-comma8comma-eight39, \quad four-three2, \end{array}$$

$$\begin{array}{c} 9\ 108 \quad 1\ 572 \quad ,\ 163 \quad 2 \quad ,\ 13 \quad 8\ 76 \quad ,\ 5\ 269 \quad ,\ 4\ 5\ 5 \quad 8\ 3\ 6 \quad ,\ 7\ 29\ r\ 6 \quad . \\ Mes \quad r\ e \left[M \quad e \right] co \quad m\ put \quad so\ n \quad yt\ h\ e \quad (2\times) \quad 31\ n \quad egra \end{array}$$

$$1\ 0 \quad nth \quad m\ 2\ d \quad l\ i\ nl \quad n\ 3\ g \quad W \quad e\ rs\ 2\ r \quad s\ fo\ r \quad m$$

$$+67 = x - 21x - 10x + 30$$

$$\begin{array}{c} o \quad h \quad m \quad d \quad n^a \quad m \\ 1^6 523, -1431489_9) \quad a_n^d \quad (10594 \end{array}$$

$$= x - 87^9 9^8 x + 3 \quad 1387_0 four-period$$

$$W\ e\ ta\ e_t\ h\ e \quad ur_v\ e$$

$$E : \quad y2$$

$$k \quad r = \quad 5 \text{ o v e r } \mathbb{Q}. \quad T \quad \text{hepoi}$$

$$(54zero-comma11eight-eight), (571836), \quad parenleft-four eight-six, 3132parenright-comma \quad (61$$

$$\begin{array}{c} r-m \quad ab \quad s-ai-s \quad o \quad theM \\ N1 =^2 1^9 0^9 a\ n \quad d\ C0 = 10600 \quad wob\ r\ noim\ p\ ov\ e\ ment\ A\ s\ n\ ab\ v-e, ^e b\ t_y \\ eim_p\ r\ oved \quad o \quad N \quad \leq 6. \end{array}$$

Ta 1 e 3

$3, 1_{0,1} 7, 376, 3 \quad 9_{,43} \text{two-comma}46\text{comma-eight}5_1 3, 52, 50, 57^{6,5} 8 \cdot 61_2 \quad 6,$
 $\text{seven-two}0, 792, 972, 172, 138, \quad 1732, 211\text{comma-three}^2 385 \quad 2\text{four-eight}, 25_9$
 $2, 368 \cdot 8$
R e een es
] A . B a ker , T e Diop h n tine e uation y 2 = ax3 + bx2 + cx + d, J . Lon d
 $c_c u_r ve : re$
 $1 _ _ 3.$
 $i_{uad} r \quad c_u^n mb \quad e^r fie \quad d_s I \quad , J N \quad u^m \quad be \quad rT h \quad e_o y \quad 3^8 1(9_{91}) 35 _ _ 51 \cdot I \quad G$
 $aan \quad d N .$
 $e_{dsI} \quad M a \quad t^h \cdot C \quad o m p \cdot 5^3(1989) 689 - 69_6$
F . . Gant ma h_{er,T} h T h_{eory} o M@ r e_{I,C} h e_l^e a N e Y
ork N.Y.19 J . G
 $c_{re}^v v \quad \mathbb{Q}, \quad i : E l i t \quad C_u r_{es} \quad n_d \quad R_e a t e d \quad T \quad o_{ic}^p, H \quad . \quad K_i s \quad l e v s \quad y \quad a n \quad d^M. \quad Ra$
M u r ($C_R \quad M P r \quad o c \quad d \quad n_{san_d} \quad L \quad c_{ture} \quad No \quad t^{es} \quad A m \quad er \quad M$
 $d_{ne}^c, R I \quad , 1 \quad 4 \quad 6 _ _ 8_3.$
D R . r_{ay} s n, Th a r t^{hmet} i geo mer
 $5 _ _ 12$
[H S A H_i n dr y a J H S v e m a n T^h e c a_{noic} a h_{egt} a n di n
e g al p o_{it_s^n} o eli tcc u e s I nv^e n^t M_{ath.93}(988)_4 19^0_{45}.
S. L a n g, D iph t ne a p_r^p o^x m a t o_n o_n t o r_{us_{es}} A_{mer} J M h.8 (6194
521 53
[L — E_{lpt^c} F c t_o ns , A_{ddi_sn}^o -W el e_y^r ea d g 1 9 3
— — — El_i
e B_{ri}, 197 — , ^C onjecture d i o_p h_{nt} i_n e t^i ma e_s o n eli_p B_{rk} e _r' B_{sel} 193, 1 —
— 551 71.
A.K.L
r t o^n a_{lc} o_{effici_n}^e ts M_{@.A}^h n.2_{61} 1982_{,5}^j 5 - 53
[M Y u I M n_{in}, C^y c_{to}^o m_{ic} f i e l s_{and} mo d_{lar} cu_r v_{es}, R_{us} i n_{@}^M h S r_{ves}^y 2
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