$afii10065 - afii10077$
— , 2009 . 50 , 3

512 . 55

SV –

. .

$afii10065 - afii10088$. R - P -
, σ(P) ,
σ(M / I (M)) -
, M — σ(P) . ,
, .

: , , SV - ,

.

, — -
. M , - , M ,
M . R -
, R_R.
J (R) J (M) -
R M . M -
E (M) . M , -
M
l (M) . , - M , M -
. R - ,
M - , σ(M) .
M SV - , V -
. R SV - , R_R — SV - .
M SV - $^{afii10070 - afii10078}$,
σ(M) . R SV - , R - .
I_0– -
[1] . . . - 90 - . .
[2] . -
1 3 ,
SV - . , SV - .
M

$$0 \subset Soc_1(\mathrm{M}) = \mathrm{Soc}\ (\mathrm{M}) \subset \cdots \subset Soc_\alpha(\mathrm{M}) \subset Soc_{\alpha+1}(\mathrm{M}) \subset ...,$$

$Soc_\alpha(\mathrm{M})/Soc_{\alpha-1}(\mathrm{M}) = \mathrm{Soc}\ (\mathrm{M}\ /Soc_{\alpha-1}(\mathrm{M}))$ -
α $Soc_\alpha(\mathrm{M}) = \bigcup_{\beta < \alpha} Soc_\beta(\mathrm{M})$ -
α. L (M) $Soc_\xi(\mathrm{M})$, ξ —

, $Soc_\xi(M) = Soc_{\xi+1}(M)$.
M , M = L (M) . R , R_R - . R L (R) Soc (R) L (R_R) Soc (R_R).
R - M - α $I_\alpha(M)$ - . $\alpha = 0$ $I_\alpha(M) = 0$. $\alpha = \beta + 1$, $I_{\beta+1}(M)/I_\beta(M)$ —
M - M /$I_\beta(M)$, - -
M - . α — , $I_\alpha(M) = \bigcup_{\beta<\alpha} I_\beta(M)$. , τ

$I_\tau(M) = I_{\tau+1}(M)$ $I_1(M/I_\tau(M)) = 0$. I (M) $I_\tau(M)$. R I (R) I (R_R), , ,
.

R - P , M S = $End_R(P)$ $Hom_R(P, M)$ S - , (fs) (m) = f (s (m)) , f $\in Hom_R(P, M)$, s $\in$ S m $\in$ M .

1 . P — R - S = $End_R(P)$. M $\in \sigma(P)$, -
:

(1) N — M , S - $Hom_R(P, M/N) \cong Hom_R(P, M)/Hom_R(P, N)$;

(2) M — R - , $Hom_R(P, M)$, S - ;

(3) M — R - , $Hom_R(P, M)$ — S - ;

(4) M = $\sum_{i \in I} N_i$ $Hom_R(P, M) \neq 0$, $i_0 \in I$,

$$Hom_R(P, N_{i_0}) \neq 0;$$

(5) $\phi \in Hom_R(P, M)$, ϕ S = $Hom_R(P, \mathrm{Jm}(\phi))$.

. (1) f — M M / N . g : $Hom_R(P, M) \to Hom_R(P, M/N)$, g (ϕ) = f ϕ. , g — S - Ker (g) = $Hom_R(P, N)$. [3 , 18 . 3] P — $\sigma(P)$. g — - , .

(2) $\phi \in Hom_R(P, M)$. , ϕ S = $Hom_R(P, M)$. $\alpha \in Hom_R(P, M)$. ϕ , . P
$\beta \in$ S , $\alpha = \phi\beta$.

(3) M = $\bigoplus_{i \in I} S_i$, i S_i . P , $Hom_R(P, \bigoplus_{i \in I} S_i) \cong \bigoplus_{i \in I} Hom_R(P, S_i)$. S - $Hom_R(P, M)$.

(4) $\phi \in Hom_R(P, M)$ — . - f $\bigoplus_{i \in I} N_i$ $\sum_{i \in I} N_i$. P —

σ(P) , g : P $\rightarrow \bigoplus N_i$, ϕ = fg . i $\in$ I g — , , $i_0 \in$ I ,

$$Hom_R(P, N_{i_0}) \neq 0;$$

(5) P . □
[4 , 1 1 . 35] .
2 . P — R - S = End_R(P) . R - M S - Hom_R(P , M) .
3 . M — R - N — -
M , (N + J (M)) / J (M) —
M / J (M) . N $afii10065 - afii10077$ mR
M , (N + J (M)) / J (M) = (m + J (M)) R .
. n — N ,
(N + J (M)) / J (M) = (n + J (M)) R .
M
mR , mR $propersubset - negationslash$ J (M) , mR ⊂ nR mR — M .
(N + J (M)) / J (M) = (m + J (M)) R ∼= mR / (J (M)∩ mR) ∼= mR / J (mR) ,
mR . □
4 . M — R - σ(M) - . - σ(M) .
. N — σ(M) . -
N , [5 , 3 . 3] , N $propersubset - negationslash$ J (E (N)) , E (N) — -
N σ(M) .
E (N) N = E (N) .
[5 , 3 . 3] J (N) , N — . □
5 . M — σ(M) , N σ(M)
.
. N , [5 , -
3 . 4] ,
N_0. M — , [6 , 3 . 1 2] N_0
, , N_0/ J (N_0) . 3 4 , N_0 - , , . □
M , M / J (M) - . , N M - M , N_1 N_2, $N_1 \oplus N_2$ = M ,
N_1 ⊂ N N_2∩ N N_2. R - M $^{afii10070-afii10078}$, M . , .

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6 . M — R - . -
:

(1) M σ(M) ;

(2) M ¨ σ(M) ;

(3) σ(M) .

. (1) $\Rightarrow$ (2) , M ¨ . N — M . , N / (N ∩ J (M)) . - N / (N ∩ J (M)) , , N —
. l (N / (N ∩ J (M))) = 1, 3 N_0 N , (N_0+ J (M)) / J (M) $\cong$ (N + J (M)) / J (M) M = N_0⊕ L , L — M . N = N_0 ⊕ (N ∩ L) , N ∩ L ⊂ J (M) J (M) ⊂ Soc (M) [5 , 3 . 3] . N ∩ L — - , 4N_0 — , N — . - S M , l (S / (S ∩ J (M))) < n N — M , l (N / (N ∩ J (M))) = n . N N_0, N_0/(N_0∩ J (M)) — - . 3 mR N_0, M = mR ⊕ L , L M . N = mR ⊕(N ∩ L)
l (N / (N ∩ J (M))) = 1+ l ((N ∩ L) / ((N ∩ L)∩ J (M))) . mR N ∩ L , , N . , M
¨ .

(2) $\Rightarrow$ (3) , σ(M) - . [6 , 3 . 1 2]
M . M / N — - M N_0 — M / N . N_0 — ¨ , - , [7 , 1 0 . 14] $N_0 = N_1 \oplus \cdots \oplus N_k$, i N_i . , , , 4 - , Soc (N_0) $\neq$ 0. , - M , M . N — σ(M) . A - N , - . , - A_0 A $\sum_{U \in A_0}$ U = $\bigoplus_{U \in A_0}$ U .

$N_0 = \bigoplus_{U \in A_0}$ U . M ¨ , [3 , 27 . 3]

N = N_0⊕ L , L — N . L — , 5 , L . N_0. , σ(M) - . [8 , 2 . 4] σ(M) .

(3) $\Rightarrow$ (1) [8 , 2.4]. □

7 . M — R - , - $\bigoplus_{\alpha \in A}$ I (M_α), M $\cong M_\alpha$ α,

M - .

. L $= (\bigoplus_{\alpha\in A}$ I $(M_\alpha))/$ N — -

$\bigoplus_{\alpha\in A}$ I (M_α) φ — $\bigoplus_{\alpha\in A}$ I (M_α) L .

α_0, $\varphi i_{\alpha_0}($ I $(M_{\alpha_0})) \neq 0$, i_{α_0} — I (M_{α_0}) $\bigoplus_{\alpha\in A}$ I (M_α). γ — -

, $\varphi i_{\alpha_0}(I_\gamma(M_{\alpha_0})) \neq 0$. , γ — . L $I_\gamma(M_\alpha)/I_{\gamma-1}(M_\alpha)$, , L M - . □

8 . M — R - , - $\sigma($ M) , :

(1) $\sigma($ M) ;

(2) $\sigma($ M / I (M)) ; (3) $\sigma($ M / I (M)) .

. (1) $\Rightarrow$ (2) (M / I (M)) / J ((M / I (M))) - M - , , , -

M - . 4 5 , M / I (M) M - , - - M - . $I_1($ M / I (M)) = 0, - . , (M / I (M)) / J ((M / I (M))) M - , , [5 , 3 . 4] . M / I (M) - 6 .

. (2) (3) [8 , 2 . 4] .

(3) $\Rightarrow$ (1) , N - $\sigma($ M / I (M)) $\sigma($ M) . E (N) — N $\sigma($ M) φ — $\bigoplus_{\alpha\in A} M_\alpha$ E (N) ,

α M $\sim= M_\alpha$. $\varphi(\bigoplus_{\alpha\in A}$ I $(M_\alpha)) = 0$, E (N) — $\sigma($ M / I (M)) . [8 , 2 . 4] E (N) — , N $\not\subset$ J (E (N)) ,

E (N) = N . , $\varphi(\bigoplus_{\alpha\in A}$ I $(M_\alpha)) \neq 0$, 7 , E (N) M - , , , E (N) = N .

N $\sigma($ M) . φ $\bigoplus_{\alpha\in A} M_\alpha$ N ,

α M $\sim= M_\alpha$. $\bigoplus_{\alpha\in A}$ I $(M_\alpha) \subset$ Ker φ,

N $\sigma($ M / I (M)) , , . $\bigoplus_{\alpha\in A}$ I $(M_\alpha) propersubset-negationslash$ Ker φ. 7 , N M - . , - , N . [5 , 3.4]. □

---

9 . P — - SV - R , S = End_R(P) — M $\in \sigma$(P) . S - Hom_R(P , M) , .

. , S - Hom_R(P , M) - . M -
α M_α . $\alpha = 0$ $M_\alpha = 0$. $\alpha = \beta + 1$, $M_{\beta+1}/M_\beta$ —
P - M $/M_\beta$. α — -
, $M_\alpha = \bigcup_{\beta < \alpha} M_\beta$. M_0

. , - α Hom_R(P , M_α) = 0. $\alpha = 0$,
. α — $\beta < \alpha$ Hom_R(P , M_β) = 0. α — , Hom_R(P , M_α) = 0 . , α
— $\alpha = \alpha_0 + 1$. Hom_R(P , M_{α_0}) = 0. 1

$$Hom_R(P , M_\alpha/M_{\alpha_0}) \cong Hom_R(P , M_\alpha)/Hom_R(P , M_{\alpha_0}) \cong Hom_R(P , M_\alpha).$$

Hom_R(P , M_α) $\neq 0$, 1 M_α/M_{α_0} P - L , Hom_R(P , L) $\neq$ 0.
2 Hom_R(P , L) — S - , Hom_R(P , M_α) , , Hom_R(P , M) P -
, . , - α Hom_R(P , M_α) = 0, , - , Hom_R(P , M_0) = 0.
M $/M_0$ P - - , [5 , 3 . 4] , M $/M_0$. -
1Hom_R(P , M $/M_0$) — , Hom_R(P , M $/M_0$) $\cong$ Hom_R(P , M) , Hom_R(P , M) . □

10 . P — R - , S = End_R(P) — N — S - . R - M , M $\in \sigma$(P) N $\cong$ Hom_R(P , M) .

. [3 , 25 . 5] , S - -
ϕ : N $\bigotimes$ S $\to$ Hom_R(P , N $\bigotimes$ P), n $\otimes$ s $\to$ [p $\to$ n $\otimes$ s (p)] , -
s s . , N $\bigotimes$ P $\in$ σ(P) . ,
s , N
Hom_R(P , N $\bigotimes$ P). N — , -
s $\phi \in Hom_R$(P , N $\bigotimes$ P) N = ϕ S . 1 ,
s N = Hom_R(P , Jm (ϕ)). □

1 1 . P — - SV - R S = End_R(P) — . S — SV - .

. [5 , 3 . 4] , - S - -
. N — S - .

1 0 , $M \in \sigma(P)$ $N \cong Hom_R(P, M)$. 9 N - . □

12 . P — SV - , P —
.
. , $P \neq L(P)$. M - $P/L(P)$, [3, 18.2] -
. [5, 3.3] , $J(M) = 0$. M - , [5, 3.4] M -
P - mR , $m \in M$. [3, 22.1, 22.2] - , $End_R(mR)$. 1 1
[5, - 3.7] $End_R(mR)$ — SV - . $End_R(mR)$ - e emR , Soc (M
) = 0. □

13 . R - P
:

(1) P — SV - ;

(2) M — $\sigma(P)$, $\sigma(M/I(M))$.

. (1) (2) 8 12. □

[8 , - 2 . 5] .

14 . R :

(1) R — SV - ;

(2) $R/I(R)$ $J^2(R/I(R)) = 0$;

(3) $R/I(R)$ -
.

15 [9] . R - P
:

(1) P — SV - ;

(2) $\sigma(P)$ P -
.

. (1) ⇒ (2) [6 , 3 . 1 2] - $\sigma(P)$. ,
$\sigma(P)$ P - .

(2) ⇒ (1) , P — SV - , , 1 2 . - , $\sigma(P)$ P - , . . P — V -
. □

SV - SV - - , . SV - ,
.

1 6 . R — , R_0 —
R , , $J^2(R_0) = 0$.

———

S = $\prod R_i$, R_i = R i . S -
i ≥ 1
T = { a ∈ S | ∃ N ∀ i , j > N : $a_i = a_j \& a_i \in R_0\}$. [5 , 1 . 2]
, Soc (T) = I_1(T) = $\bigoplus R_i$.
i ≥ 1

N — T / Soc (T) - , T - . N ⊂ E (N) , E (N) — T - N . E (N) Soc (T) ≠ 0, e Soc (T) E (N) e ≠ 0. N E (N) e — - T , Ne ≠ 0, N Soc (T) = 0. - , E (N) Soc (T) = 0 , , E (N) T / Soc (T) . N — T / Soc (T) - , N = E (N) .

, , T / Soc (T) , - T . , I_2(T)/I_1(T) = I_1(T / Soc (T) T / Soc (T)), , , I (T) = I_2(T) = { a ∈ S | ∃ N ∀ i , j > N : $a_i = a_j \& a_i \in I_1(R_0)\}$
T / I (T) $\cong$ R_0/ I (R_0). 1 4 , S SV - . , , R = M_2(P) , R_0 — P , [1 0] , .

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