KYBERNETIKA — VOLUME 48 (2012) NUMBER 5 PAGES 958–967

CHANCE CONSTRAINED BOTTLENECK TRANSPORTATION PROBLEM WITH PREFERENCE OF ROUTES

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This paper considers a variant of the bo tt l − ee − nc − k t a − r n s − p ortati n − o problem Fo eac supply demand point pair , the transporta t i − o n tme i an n − i d e − pn − e den r − a n d m − o varia· bl e Preferenc of each route is attached . Our model has two criteria nam el y minimiz th transportatio time target subj ect to a chance cons t − r ain and max , imiz th minima preferenc amon th used routes . Since usually a transporta t i − o n pa tte n − r o ptimiz i − n g tw ob[1] jective simultaneousl does not exist , we define non - domina t i − o n n − i thi sett i − n g a d − n propos a efficien algorit h − m t find some non - dominated transporta t i − o n pa tterns We th n − e s − hw − o th ntim complexit o th proposed algorithm . Finally , a nume r i − c al examp e − l.i pre s − e nt d − e t illustrat e$_{ho-w}$ ou algorit m − h works .

Keywords : bottleneck transporta ti on , random t r − a n p − s ortati o − n time chanc constrain t
preference of routes , non - dom i − n a t o − i n
Classification : 90 C 35 , 90 C 1 5 , 90 C 70 , 68 Q 2 5

1 . INTRODUCTION

The classical transportation problem i − s defined by minimizat o − i n o v ariabl transporta tion costs while meeting a set of demands f r − o m a s e of av ai a − l bl e s − u p plie s I ti sals oknow as the cost minimizing transporta t i − o n pro b e − l m w t$_{hi-c}$ h h a be n − e extensivel studie i the lit erature and several algor i − t hms four − bracketleft,6, 7 1,$_2$ 1 417 ar av ai l − a bl t solv i t d$_{On}$ well studied variant of the classical transpo rtat i − o n p o − r *blm* e i *k* n o − w n a o th bottlenec transportation problem , which determines a s i − n g e − l bo t tl e − n e k − cs$_{fo}$ th transportatio nsys tem by minimizing the maximum time for t − r an p − s or of a l good s e$_I$ i als know a the time minimizing transportation prob l − e m iS milarly ma y − n effi cien· algor^o ithm n$_{hav}$ been proposed by Garfinkel and Rao [8] Hammer bracketleft − one1] R ussel e a l [16] Srinivasa nan
Thompson [1 8] and Szwarc [1 9] for so , lvi − n g i t As f o s o − t c hast[1] i progr^{a-m} m in g th basi properties and algorithms [1 5] the sta b il i − t$_y$ and rob ustnes [3 a d − n th c − h$_a$ nc econstrain condition [1 3] are well studied . Recen tly Adey e − f$_a$ and Lu s$_{a-hn}$ d ju l − a w [1 presente th main principle of multiobjec ti ve sto chas , ti − c l n − i e a p r − o g a − r m m i − n g St ochasti version o bottleneck transportation prob l − e m are conside e − r d [9 1 0] Ge a n − d Ishi [9 propose a sto chastic bottleneck transporta t i − o n prob e − l m wi t − h f uz y − z s − u p p y − l a n − d d m − e an d Geeth and Nair [1 0] presented a single c r i − t$_e$ r o − i n bo tt l − e e − nc − k t a − r n s − p ortati n − o probl e − m w it ran

dom transportation cost . Be si des , Chen et a l [5 s t − u d e − i d a fuz z − y t a − r nsportatio nprobl e − m with preference of routes .

The model considered in th i − s paper s − i an e x t − e ns o − i n of thes p reviou model s We extend the bottleneck transporta t i − o n pro b e − l m by con s i − d er n − i g r − a n e dm nes o ftransporta tion t ime and preference of route . Randomne s − s mea n h − t a th t r − a nsportatio ntim ema change according to many factors . The pr e − fr − ee − n ce of o − r ut reflect th degre o satis faction with respect to the chosen route . So t − w o criter a − i a r t − a k n − es int accoun t f On i to minimize the transportation t ime target su b j−e c o − t a c h − a nc o − c nstra o in t T h othe ri to maximize the minimal preference among t − h e u e − s d r − o ute s B u usuall a transporta tion pattern optimizing two objec t i − v es simu laneo us y − l d oe n o exis t S we see som non - dominated transportation patterns

The rest of this paper is organ i − z ed as f ol o − l w s O u p r − o b e − l m i fo r − m ulat d − e i − n Sectio n 2 and then in Section 3 we present an effic i − e nt a g − l ori t − h m o − tfi n − d som n on - dominate transportation patterns . Section 4 shows how o u a l − g ori h − t m w ork u esin a numerica example . Finally , Section 5 concludes t h i − s paper and di c − s usse furthe rresearc hproblems

2 . PROBLEM FORMULATION

In this paper , we consider the fo l − l own−ig bi hyphen − c ri e − t r i − a bo ttlene k − c t r − a nsportati n − o probl e − m with randomness of transportation time and p r − e fe e − r n − c e of o − r ut e

(C 1) There exist m supply points $\{S_1, ...\ S_m\}$ and n dma n − d p oint $\{T_1, ..., Tn\}$ Th total upper limit provided from each supp l − y p oin e$_S$ i a a d − n th tota llowe rlimi to each demand point T_j i − s jb. Furthe r we as u − s me t − h a thes a_i jb ar positiv

$$\text{integersand} \sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j .$$

(C 2) Let (i, j) denote the route from supp l − y p oin S o − t dma n − d p oin Tj = $1, ..., m$ $j = 1, ..., n$. Preference of route i − s attac e − h d and i i as s − u m d − e t be measure b

a real number μ_{ij} between 0 and 1 . T h i − s number e − rt flect th degre o $fsatisfact$ d io

with respect to the chosen route . The v a l − u e $\mu ij = 1$ corresp n−o d t complet sat isfaction , while $\mu_{ij} = 0$ corresponds to comp le t − e dissatisfactio n s Fo intermediat numbers , a higher value corresponds to a h i − g he de gre of satisfactio n

(C 3) For each route (i, j), the transporta t i − o n tme tij i an n − i dep e−n den tr n − a d o − m variabl

according to Wedenote athenormaldi − stributransportation quan t o − i n $N_{\text{ti}-\text{t}_\text{y}}(m^{\sigma^2{}_{ij})}_{ij_{\text{usn}-\text{i}}}$ g wt − h e i h − t r−o muta − en$_{(i}, j^{mij}$ that these x_{ij} are nonnegative decis i − o n v ar a − i ble s The followin chanc constrain is attached :

$$\Pr\{ti - j \leq f\} \geq \alpha \quad (i, j) \mid xij > 0 \tag{1}$$

where $\alpha > 0.5$ and f is also a decis i − o n v ar a − i b l − e d n − e ot i − n g th targe o bottlenec transportation time to be minim i − z ed

(C 4) We consider two criteria : one i − s to ma xm z − i e th min i − m a preferenc amon th used routes and the other i − s to minm z − i e f

Under the above setting , our chance con s − $t_{\text{ran}-i}$ ed bo tt l − e ne k − c t r − a nsportatio probl m − e
with preference of routes can be formula t − e d as f ol o − l w s

$$\text{TP}: \quad \text{minimize} \quad f$$

$$\text{maximize} \quad \min_{icomma-j} \{\mu_{j-i} \mid x_{ij} > 0\}$$

$$\text{subject to} \quad \Pr\{ti - j \leq f\} \geq \alpha \quad (ij) \mid xij > \quad 0 \quad i = \quad 1, ..., mj = \quad 1, ...n$$

$$\sum_{j=1}^{n} x_{ij} \leq \quad ai, \quad i = 1, \quad m$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j, j = 1 \quad n$$

$$x_{ij}: \quad \text{nonnega t i − v e} \quad i = 1 \quad m \quad j = \quad 1, ..., n$$

In order to solve problem TP , first we in t − r oduce h − t_e fol l − o w n − i g e − q u v − i alen parametri programming formulations .

The chance constraint (1) reduces to

$$F(\frac{f - m_{ij}}{\sigma_{ij}}) \geq \alpha \quad (ij) \mid xij > 0$$

where $F(\cdot)$ is the cumulative d i − s t rbu t o − i n func t i − o n of th s a − t n − d a d − r n o r − m a distributio $N(0,1)$. That is ,

$$(1) \iff \frac{f - m_{ij}}{\sigma_{ij}} \geq K_\alpha \quad (i,j) \mid xij > 0$$

$$\iff f \geq m_{ij} + K_\alpha \quad \sigma ij \quad (i,j) \mid xij > 0$$

$$\text{where} K_\alpha = F^{-1}(\alpha).$$

Since f should be minimized , then pro b l − e m TP e − r du ce t o

$$\text{P}: \quad \text{minimize} \quad \max_{icomma-j} \{m_{ij} + K_\alpha \quad \sigma ij \mid xij > 0\}$$

$$\text{maximize} \quad \min_{icomma-j} \{\mu_{ij} \mid xij > 0\}$$

$$\text{subjectto} \quad \sum_{j=1}^{n} x_{ij} \leq \quad ai \quad i = 1 \quad m$$

$$\sum_{i=1}^{m} x_{ij} \geq \quad bjj = \quad 1 \quad n$$

$$x_{ij}: \quad \text{nonnega t i − v e} \quad i = \quad 1 \quad m \quad j = \quad 1, ..., n$$

Next , we define the bi - objective vec t − o r v (**x**) of a t a − r n s − p ortati n − o patter **x** = $(xij$
feasible for P as

$$\text{v}(\mathbf{x}) = (v(\mathbf{x})1, v(\mathbf{x})2) = (\max i,j \quad \{m_{ij} + K_\alpha \quad \sigma ij \mid xij > 0 \quad\} \text{ m } \ddot{\imath}\text{in}_j \{\mu ij, ^{|x} ij > \quad 0\}$$

Generally , a transportation pattern optimiz i − n g w − t o o b jective ss i − m ul a − t neousl ydoe sno

exist . Therefore , we seek some non - dom i – n a e – t d t r – a n p – s ortat i – o n p a ttern s th definitio no
which is given as follows .

Definition 2 . 1 . Let $\mathbf{x}^a, \mathbf{x}^b$ be two transpo r $-$ t$_a$ t o $-$ i n pa ttern t $-$ h a ar feasibl efo rP . The n we say that $\mathbf{x}^a$ dominates $\mathbf{x}^b$, i $-$ f$v(\mathbf{x}^a)1 \le v(\mathbf{x}^b$one $-$ parenright $v$parenleft $-$ x$a) \ge v\mathbf{x}$ $-$ **parenleft**$b)$ a n $-$ d $(v\mathbf{x}$ $-$ **parenleft**$a)_1, v$parenleft $-$ x$a)2 \neq (v(\mathbf{x}^b)1, v(\mathbf{x}^b)2)$. If there ex i $-$ s ts no transpo rtat'o $-$ i n pa^2 tte r $-$ n do 2mn $-$ i atin **x x** i calle a non - dominated transportation pattern .

3 . SOLUTION PROCEDURE Sorting $\mu_{ij}, i = 1, ..., m, j = 1, ..., n$, and le the resul be

$$0 < \mu^1 < \quad < \mu^g \le 1$$

where g is the number of different values of them

Compute $m_{ij} + K_\alpha \sigma_{ij}$, $i = 1, ..., m$ $j = 1$ $, n$ a n $-$ d ar a $-$ r ng thes value i
ascending order . Let the result be

$$c^1 < \quad . \quad < c$$

where l is the number of different values of them Let $\mathbf{C}$ $\triangle_=$ $(mij + K_\alpha \quad \sigma ij_{\text{parenright}-\text{m}} \quad \times n$

For$u = 1, ..., g, k = 1, ..., l,$ set

$$c^{u,}ij^k = \{ \quad M^0 \quad \text{otherw}^{\text{if}\mu i-j\ge\mu^u,}_{\text{s}-\text{i} \qquad \text{e}} \quad m_{ij} + K_\alpha \quad \sigma ij \le c^k \quad i = \quad 1 \quad . \quad .m \quad j = \quad 1, ..., n$$

where M is a sufficiently large value .

For $u = 1, ..., g, k = \quad 1, ..., l,$ deno t $-$ e the c os minimiz n $-$ i g t r $-$ a nsportatio probl m $-$ e with the above defined cost values as P_u^k :

$$\mathrm{P}_u^k: \quad \text{minimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} cuij^{comma-k} \quad xij$$

$$\text{subjectto} \quad \sum_{j=1}^{n} xij \le \quad ai, \quad i = \quad 1 \quad m$$

$$\sum_{i=1}^{m} xij \ge \quad bjj = \quad 1 \quad n$$

$$x_{ij}: \quad \text{nonnega t i} - \text{v e} \quad i = \quad 1 \quad .m \quad j = \quad 1, ..., n$$

For fixed $u \in \{1, ..., g\}$ and $k \in \{1, ... \quad l\}$, no t $-$ e h $-$ t$_@\mathrm{P}_u^k$ i a restrict e $-$ d transportatio problem , therefore it is not always ha v $-$ e a fea sib e $-$ l solut i $-$ o n w i h $-$ t opt i $-$ m a valu 0 I it exists a feasible solution , then i $-$ ts $-$ i fea sib l $-$ e on y $-$ l u s i $-$ n g th rout $(i, j$ wit $\mu i \quad \ge$

$$m_{ij} + K_\alpha \sigma_{ij} \le c^k. \qquad \mu^u,$$

Denote

$$S(u) = \{(i,j) \mid \mu_{ij} = \mu^u, i = 1, \quad m \quad j = \quad 1 \quad n \quad \} \quad u = \quad 1, ...g$$

$$T(k) = \{(i,j) \mid m_{ij} + K_\alpha \sigma i - j = c^k, i = 1 \quad m \quad j = \quad 1 \quad . \quad ., n\} \quad k = \quad 1, ...,$$

$$p = \max \left\{ tvextendsingle \sum_{=r}^{g} \quad S(\quad r) \ge n \right\}$$

$$q = \min \left\{ t \;\middle|\; \sum_{r=1}^{t} T(r) \geq n \right\}$$

It is obvious that P_u^k is infeasib l – e when $u \in \{p+ \quad 1 \quad g\}$ or $k \in \{1, ..., - \quad 1\}$

such For that each $P_u^{k_u} \in$ is $\{1,...,p\}$, we feasible. If such then k give the exi–ss–t, al – godeno ri e–ti t–hm by k_u .o–tfi n – d th m – salles Otherwise se $k_{th} \in$

Remark 3 . 1 .

Remark 3 . 1 . If exists $u_0 \in \{1, ..., p\}$, such that $P_{u_0}^k$ i infeasibl fo a n $k \in \{q, ..., l\}$ then $P_u^k, u = u_0 + 1, ..., p$ are also infeasib l – e f o any $k \in \{ \quad q \quad . \quad ., l\}$

For each $u \in \{1, ..., p\}$, we need to find the sma lles k_u su c – h h – t a P_u^k i feasibl e

The smallest k_u corresponds to the b i – g ge t – sc^{k_u} w h i – ch t i th firs componen o th

bi - objective vector (c^{k_u}, μ^u). The m an idea t – o find h – te sma lles t$_k$ su c – h tha f$_P^k$ i feasible is based on a binary method , w h i – c h s – i g i – vn – e as fol l – o w n – i t detail

Algorithm (To find the smal l – e st k such that P_1^ks – i feasible

Step 1Set$L$ = setting$k_1^q$and$_{=L}$.check Otherw whether i–s e , $P_{setU}^{L_1}$s – i=fea$_{land}$sib$_c^{l-e}$or$_{e-hc-k}$n$_w^{ot}$.hethe I feasibl$_{U_P}$ i

e ter $feasi$ minat afte bl o not

If feasible , go to Step 2 . Otherw i – s e e – t rm i – n a e – t d u t – o infeas[1]ibil[s] ity

Step 2 When $U - L > 1$, set $K = \lfloor (L+U)/2 \rfloor$ and c h – ek – c w hethe P_1^K i feasibl eo rno t where $\lfloor \cdot \rfloor$ denotes the greate s – tn – it – e g r – e not g r – e ate h – ta – n · r I feasi[s] bl e se $U = K$

and repeat Step 2 . Otherw i – s e , s e $L = K$ and repea St p – ef$_2$ W he $U - L =$ 1 go to Step 3 .

Step 3 If P_1^L is feasible , set $k_1 = L$. Other w i – s e se $k = U$

For $P_u^k, u = 2, ..., p$, the algo r i – t hm s – i v e – r y simila o – t t – h a o k_{P1} th onl differenc is we first set $L = k_{u-1}$.

Denote $A = \{(u, k_u) \mid k_u$ ex i – s ts $, u = 1, \quad . \quad p\}$ f – I t – h er exist $(u_1, k_u1$ an $(u_2, k_u2$ $\in A$ such that $u_1 \neq u_2$ but $k_{u_1} = k_{u_2}$, then d ele t – e (m n – i{ $u_1, u2\}k$ mi n $\{u_1, u2\}$ fr o – m A Let obtained set after deletion be B. No t – e that a l e lm ent n – iB hav differen firs components and also different second componen t s

For all $(u, k_u) \in B$, solve prob l – e ms $P_u^{k_u}$ ' s and le d n – e ot th o pt i – m a transportatio patterns by $u_{\mathbf{x}}^{k_u}$ ' s . Then we find a set of some non hyphen – d$_o$ m n – i at e – de$_{ta-r}$ nsportatio pattern and that of the corresponding bi - o b^{j-e} c t v – i e vec tor of p r – o blm P denot d – e b NDT an NDV respectively .

The validity of our solu ti on procedure i – s shown n – i t – he follow n – i g p ropositi o – n

Proposition 3 . 2 . The solution procedure f o – r P s – i v ali d

method.Foreach$(u^{(u,k) \in A}$. Fr–oeach binaryfeasibilitychecking Proof. eachfeasibleP_u^k, wehave Foreach$u$,the algo ri–thmtofind t–he $k_u) \in A$ smalles P_u^k i$_k^B$ an suc–h feasi t ble

$i \setminus j$	1	2	3	a
1	$N(3, 0.5^2)$	$N(70period-four^2_)$	$N(4$ 1two − period2	5
2	$N(6, 0.8^2)$	$N(50period-three^2_)$	$N(1,0$seven − period2	8
3	$N(7, 0.3^2)$	$N(40period-six^2_)$	$N(8$ 1zero − period2	3
b_j	60	35	55	–

Tab . 1 The values of ai jb and t − h$_e$ distr b − i ut i − o n of tij

pattern $u_{\mathbf{x}}^{k_u}$ of problem P_u^{ku} is a non - dom i − n a e − t d t − r a n p − s o r tat i − o n p atte n − r o fprobl e − m P an (c^{k_u}, μ^u) is the corresponding biob j − e c t v − i e vec to r t − h$_@$ i s $NDT = \{\mathbf{x}^k u \mid (u, ku \in B \}$ $NDV = \{(c^{k_u}, \mu^u) \mid (u, k_u) \in B\}$. There f − o re , our s olut o − i n pr oc e − d ur i vali d □

Next we show the time complex i − t$_y$ of our s olut o − i n pr oc e − d ur fo P

Theorem 3 . 3 . The time complex i − t$_y$ of our s olut o − i n pr o c − e dur fo P i

$$\mathrm{O}(mn(m+n)^3 \mathrm{l-og}(m+n))$$

P r o o f . Note that $g = l = \mathrm{O}\ (mn)$, so so rt i − n g μij and $mij + K_\alpha$ σij bo h − t take a

mostfindthe$^{\mathrm{O}(mn\log(mn))}_{\mathrm{smallest}k\in}$operations$\{_{q,\ldots,l\}\mathrm{such}}For_{\mathrm{that}}k^u_{\mathrm{each P}u}$t − h$^{\mathrm{e}}_{\mathrm{ife}}$tmeasi$_{\mathrm{i}}bl^{\mathrm{comp}}$folo − llexi$^{\mathrm{t-y}}_{\mathrm{w}}$ $_{\mathrm{frm-o}}$o$_{\mathrm{fth}}$thalgorit$^{\mathrm{h-m}}_{\mathrm{face}}$ $_{\mathrm{tha}}$t

binary search over l values has time comp l − e xi y − t O (l − o g l) and ea h − c feasibilit check ing takes O (mn) because at most O (mn) e l − e men t h − s ou l − d b checked S fo eac u

checking$_{\mathrm{@most}}$totally$^{\mathrm{needs}}_{\mathrm{O}(g)}$ $_{\mathrm{times}}$O$^{(mn\ \log(mn)}_{\mathrm{tofindthe}}$$^{\mathrm{compu}}_{\mathrm{sma)ll-e}}$$_{\mathrm{t-s}}k$ $\in^{\mathrm{tat\ i-onal}}_{\{q}$ tm − i$_{,l-braceright}$e$^{\mathrm{sThe}}_{\mathrm{su}}$ $_{\mathrm{h-ct-h}}$a$_{\mathrm{a}}$lgori$^{\mathrm{h-tm}}_{\mathrm{P}^k_u\ \mathrm{i}}$feai$^{\mathrm{i}}_{\mathrm{s}}$ $^{\mathrm{exe}}_{\mathrm{ble}}$

checking totally needs O $((mn)^2 \log\ (mn))$ So lv i − n g e c − a h feasibl classica transportatio problem takes at most O $((m+n)^3 \mathrm{l-o\ g}\ (m+n)$ (se [2] a d − n e total l − y at m os O $m-parenleft$ n classical transportation problems should be s o l − v e d e h − t erefor thi p ar take a mos O $(mn(m+n)^3 \log\ (m+n))$ computa t i − o nal tm e s Co ns q − e uently th tim complexit i O (max $\{(mn)^2 \log\ (mn), mn(m+n)^3 \mathrm{l-o\ g}\ (m+n)\}) = \mathrm{O}\ (mn(m+n)$ 3 log $m-parenleft+n))$ □

4 . NUMERICAL EXAMPLE

Consider problem TP with $\alpha = 0.9987$, $tij \sim N(m_{ij},^2_{\sigma\ ij}$ a n − d th v alue o a_i b ar given in Table 1 . The preference of rout e − s a e − r g i − vn − e i − n th fol o − l w i − n g m atr$^{\mathrm{s}}$ i x

$$\mathbf{U} = \begin{pmatrix} 0.5 & \mathrm{0eight-period} & \mathrm{0four-period} \\ 0.75 & 0period-six & 0period-seven \\ 0.85 & 1 & \mathrm{0six-period} \end{pmatrix}$$

 Our problem TP reduces to prob l − e m P :

$$P: \quad \text{minimize} \quad \max_{i,j} \{m_{ij} + 3period - zero \quad \sigma ij \mid xij > 0\}$$
$$\text{maximize} \quad \min_{i,j}\{ \quad \mu ij \mid xij > 0\}$$

$$\text{subject to} \quad \sum_{j=1}^{3} xij \leq ai \quad i = 1,23$$

$$\sum_{i=1}^{3} xij \geq bj j = 1,23$$

xij nonn e − g a tiv e $ij, = 1,2,3$ Sorting $\mu_{ij}, i, j = 1,2,3$, we obtain

$$0 < \mu^1 = 0.4 < \mu^2 = 0.5 < \mu^3 = 0period - six < \mu^4 = 0\text{seven} - \text{period} < \mu^5 = 0.7$$
$$< \mu^6 = 0.8 < \mu^7 = 0period - eight5 < \mu^8 = 1$$

Compute $m_{ij} + 3.0\sigma_{ij}, i, j = 1,2,3$, we o bta i − n

$$\mathbf{C} = \begin{pmatrix} 4period - five & 8period - two & 7\text{six} - \text{period} \\ 8\text{four} - \text{period} & 5\text{nine} - \text{period} & 3period - one \\ 7\text{zero} - \text{period} & 5period - eight & 11\text{zero} - \text{period} \end{pmatrix}$$

Arrange these values in ascending orde r that i s

$$c^1 = 3.1 < c^2 = 4.5 < c^3 = 5period - eight < c^4 = 5\text{nine} - \text{period} < c^5 = 7period - six < c^6 = 7.$$
$$< c^7 = 8.2 < c^8 = 8period - four < c^9 = 1 \quad 1.0$$

For $u = 1, ..., 8, k = 1, ..., 9$, set

$$c^{u,k}ij = \{ \quad M^0 \quad \text{otherw}^{\text{if}\mu i - j \geq \mu^u}_{\text{s−i} \quad \text{e}} \quad m_{ij} + 3period - zero \quad \sigma ij \leq c^k \quad ij, = 1,2,3$$

where M is a sufficiently large value .

It is obvious that $p = 6, q = 3$.

For $u = 1, ..., 6, k = 3, ..., 9$, prob l − e m P_u^k has h − t$_e$ fol l − o w n − i g fo r − m

$$P_u^k: \quad \text{minimize} \quad \sum_{i=1}^{3}\sum_{j=1}^{3} cuij^{comma-k} \quad xij$$

$$\text{subject to} \quad \sum_{j=1}^{3} xij \leq ai \quad i = 1,23$$

$$\sum_{i=1}^{3} xij \geq bj j = 1,23$$

$$xij \quad \text{nonn e − g a tiv e} \quad ij, = 1,2,3$$

Next we give the solution procedure for p r − o b e − l m P

Find the smallest $k \in \{3,...,9\}$ **such that** $\mathbf{P}_1^k$ **i − s f a − e sible Step 1 .** Set $L = 3$ and P_1^3 is infea sib l − e. Set $U = 9$ and P_1^9 i feasibl e G t Ste 2 **Step** 2. $U - L = 6 \neq 1$. Set $K = 6$ and P_1^6 i − s fea sibl e S e $U =$ 6 repea Ste 2 **Step** 2. $U - L = 3 \neq 1$. Set $K = 4$ and P_1^4 i − sn − if − e a sibl e Se $L =$ 4 repea Ste 2 **Step** 2. $U - L = 2 \neq 1$. Set $K = 5$ and P_1^5 i − sn − if − e a sibl e Se $L =$ 5 repea Ste 2

Step 2. $U - L = 1$, so go to Step 3 .

Step 3.P_1^5 is infeasible , so set $k_1 = 6$. **Find the smallest** $k \in \{6,...,9\}$ **such that** $\mathbf{P}_2^k$ **i − s f a − e sible Step 1 .** Set $L = 6$ and P_2^6 is feasib l − e. Set $k_2 = 6$ **Find the smallest** $k \in \{6,...,9\}$ **such that** $\mathbf{P}_3^k$ **i − s f a − e sible Step 1 .** Set $L = 6$ and P_3^6 is infea sib l − e. Set $U = 9$ and P_3^9 i feasibl e G t Ste 2 **Step** 2. $U - L = 3 \neq 1$. Set $K = 7$ and P_3^7 i − sn − if − e a sibl e Se $L =$ 7 repea Ste 2 **Step** 2. $U - L = 2 \neq 1$. Set $K = 8$ and P_3^8 i − s fea sibl e S e $U =$ 8 repea Ste 2

Step 2. $U - L = 1$, so go to Step 3 . **Step** 3.P_3^7 is infeasible , so set $k_3 = 8$. **Find the smallest** $k \in \{8,9\}$ **such that** $\mathbf{P}_4^k$ **i − s fea sibl e Step 1 .** Set $L = 8$ and P_4^8 is feasib l − e. Set $k_4 = 8$ **Find the smallest** $k \in \{8,9\}$ **such that** $\mathbf{P}_5^k$ **i − s feasible**

existsno$^{\mathrm{forP}_6^k.}_{\mathbf{Step1}.}$ $^{\mathrm{Set}}kL^{=}_{\in\{8,}9^8$and}such$^{\mathrm{P}_5^8}$that$^{\mathrm{isinfeasib}}\mathrm{P}_5^k$ i − se − l.feaSet$^U_{\mathrm{si-bleF}} = 9$o − r$_\mathrm{m}and_\mathrm{Rema}\mathrm{P}_5^9$ ir − kinfeasibl$_{3.1}$ suh-

Therefore $B = \{(2,6),(4,8)\}$. So l − v e P_2^6 and P_4^8 we o bta n − i th o p t i − m a transportatio patterns $2^6_{\mathbf{x}}$ and $4^8_{\mathbf{x}}$, respectively :

$$2^6_{\mathbf{x}}: \quad x_{11} = 50, \quad x_{22} = 15, \quad x_{23} = 55, \quad x_{31} = 1 \quad 0 \quad x3 = 2 \quad 0 \quad \text{othe} \quad xij = \quad 0$$

$$4^8_{\mathbf{x}}: \quad x_{12} = 35, \quad x_{21} = 30, \quad x_{23} = 55 \quad x_{31} = 3 \quad 0 \quad \text{othe} \quad xij = \quad 0$$

which are the non - dominated transporta t i − o n pa t e − t r n of p r − o blm P a d − n th correspond ing bi - objective vectors are (7 . 9 , 0 . 5) and $(8period - four, 0.7)$ respectively T ,ha i s

$$NDT = \{\mathbf{x}_2^6 4^8_{\mathbf{x}} \quad \}$$

$$NDV = \{(7period - nine0.5) \quad (8.4, 0.7)\}$$

5 . CONCLUSION

In this paper , we have considered a bi - c riter i − a cha c − n e c − o nstrain d − e bottlenec trans portation problem with preference of rout e − s and de ve o − l p e − d an a lgori$^{\mathrm{h-t}}$ m t fin som non - dominated transportation pat t − e rns . Fur h − t$_\mathrm{e}$ r we ha v s − hw − o n th validit an tim complexity of the algorithm . Be si des our a l − g o ri'h − t$_\mathrm{m}$ i i$^\mathrm{e}$ llustrat e − d u sin a numerica

example . As a further research prob l – e m , we shou d – l c onside th flexibilit yo suppl an demand quantity , which is the case that the tota qua nti t – y frm supplie fi les y tha that to demand customers . Th i – s case mak s – e t – h$_e$ p o – r b l – e m thre criter a – i on an we ar now attacking this case . Addit i – o na lly the e – r rema i – n ma y – n o the v ariant o bottlenec
transportation problem to be considered and s o l – v ed

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