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# A GENERALIZATION OF FIXED POINT THEOREMS IN $S-$ METRIC SPACES

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**Abstract .** In this paper , we introduce $S-$ metric spaces and give some of their properties .

Also we prove a fixed point theorem for a self - mapping on a complete $S-$ metric space .

## 1 . Introduction

Metric spaces are very important in mathematics and applied sciences . So , some authors have tried to give generalizations of metric spaces in several ways . For example , G ä hler [ 3 ] and Dhage [ 2 ] introduced the concepts of 2 - metric spaces and $D-$ metric spaces , respectively , but some authors pointed out that these attempts are not valid ( see [ 6 – 1 0 ] ) .

Mustafa and Sims [ 4 ] introduced a new structure of generalized metric spaces which are called $G-$ metric spaces as a generalization of metric spaces $(X, d)$ to develop and introduce a new fixed point theory for various mappings in this new structure . Some authors [ 1 , 5 , 1 3 ] have proved some fixed point theorems in these spaces .

Recently , Sedghi et al . [ 1 2 ] have introduced $D^{*}-$ metric spaces which is a prob - able modification of the definition of $D-$ metric spaces introduced by Dhage [ 2 ] and proved some basic properties in $D^{*}-$ metric spaces , ( see [ 1 1 , 1 2 ] ) .

In the present paper , we introduce the concept of $S-$ metric spaces and give some of their properties . Then a common fixed point theorem for a self - mapping on complete $S-$ metric spaces is given .

We begin with the following definitions :

Definition 1 . 1 . [ 4 ] Let $X$ be a nonempty set and $G : X \times X \times X \to [0, \infty)$ be a function satisfying the following conditions for all $x, y, z, a \in X$,

$$G(x, y, z) = 0 \text{if} x = y = z, \tag{G1}$$

( G 2) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,

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 ( G 3) $G(x,x,y) \leq G(x,y,z)$ for all $x,y,z \in X$ with $x \neq y$,

$$G(x,y,z) = G(x,z,y) = G(y,z,x) = \cdots, \tag{G4}$$

( G 5) $G(x,y,z) \leq G(x,a,a) + G(a,y,z)$ for all $x,y,x,a \in X$.

Then the function $G$ is called a *generalized metric* or a $G-$ *metric* on $X$ and the pair

$(X,G)$ is called a $G-$ metric space .

We can find some examples and basic properties of $G-$ metric spaces in Mustafa and Sims [ 4 ] .

Definition 1 . 2 [ 1 2 ] Let $X$ be a nonempty set . A generalized metric ( or $D^*-$ *metric* ) on $X$ is a function : $D^* : X^3 \to \mathbb{R}^+$ that satisfies the following conditions

foreach$x,y,z,a \in X$.

(1) $D^*(x,y,z) \geq 0$,

(2) $D^*(x,y,z) = 0$ if and only if $x = y = z$, (3) $D^*(x,y,z) = D^*(p\{x,y,z\})$, ( symmetry ) , where $p$ is a permutation function ,

(4) $D^*(x,y,z) \leq D^*(x,y,a) + D^*(a,z,z)$.

The pair $(X,D^*)$ is called a generalized metric ( or $D^*-$ metric ) space . Immediate examples of such functions are :

$$\text{(a)}D^*(x,y,z) = \max\{d(x,y), d(y,z), d(z,x)\},$$
$$\text{(b)}D^*(x,y,z) = d(x,y) + d(y,z) + d(z,x).$$

Here , $d$ is the ordinary metric on $X$. ( c ) If $X = \mathbb{R}^n$ then we define

$$D^*(x,y,z) =|| x+y-2z || + || x+z-2y || + || y+z-2x || .$$

( d ) If $X = \mathbb{R}^+$ then we define

$$D^*(x,y,z) = \{0_{\max}\{x,y,z\} \quad \text{otherwise}^{\text{if}x=y=}_{,} z,$$

Remark 1 . 3 . It is easy to see that every $G-$ metric is a $D^*-$ metric , but in general the converse does not hold , see the following example .

Example 1 . 4 . If $X = \mathbb{R}$, we define

$$D^*(x,y,z) =| x+y-2z | + | x+z-2y | + | y+z-2x | .$$

It is easy to see that $(\mathbb{R}, D^*)$ is a $D^*-$ metric , but it is not $G-$ metric . Set $x = 5$, $y = -5$ and $z = 0$ then $G(x,x,y) \leq G(x,y,z)$ does not hold .

Now , we introduce the concept of $S-$ metric spaces which modifies $D-$ metric and $G-$ metric spaces .

## 2 . S - metric spaces

We begin with the following definition .

Definition 2 . 1 . Let $X$ be a nonempty set . An $S-$ *metric* on $X$ is a function
$S : X^3 \to [0, \infty)$ that satisfies the following conditions , for each $x, y, z, a \in X$,

$$S(x, y, z) \geq 0, \tag{1}$$

(2) $S(x, y, z) = 0$ if and only if $x = y = z$,

$$S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a) \tag{3}$$

The pair $(X, S)$ is called an $S-$ *metric space* .

Immediate examples of such $S-$ metric spaces are :

( 1 ) Let $X = \mathbb{R}^n$ and $\| \cdot \|$ a norm on $X$, then $S(x, y, z) = \| y + z - 2x \| + \| y - z \|$ is an $S-$ metric on $X$. ( 2 ) Let $X = \mathbb{R}^n$ and $\| \cdot \|$ a norm on $X$, then $S(x, y, z) = \| x - z \| + \| y - z \|$ is an

$$S - \text{metricon} X.$$

( 3 ) Let $X$ be a nonempty set , $d$ is ordinary metric on $X$, then $S(x, y, z) = d(x, z) + d(y, z)$ is an $S-$ metric on $X$.

Remark 2 . 2 . It is easy to see that every $D^*-$ metric is $S-$ metric , but in general the converse is not true , see the following example .

Example 2 . 3 . Let $X = \mathbb{R}^n$ and $\| \cdot \|$ a norm on $X$, then $S(x, y, z) = \| y + z - 2x \| + \| y - z \|$ is $S-$ metric on $X$, but it is not $D^*-$ metric because it is not symmetric .

Example 2 . 4 . [ intuitive geometric example for $S-$ metric ] Let $X = \mathbb{R}^2$, $d$ is an ordinary metric on $X$, therefore , $S(x, y, z) = d(x, y) + d(x, z) + d(y, z)$ is an $S-$ metric on $X$. If we connect the points $x, y, z$ by a line , we have a triangle and if we choose a point $a$ mediating this triangle then the inequality $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$ holds . In fact

$$\begin{aligned} S(x, y, z) &= d(x, y) + d(x, z) + d(y, z) \\ &\leq d(x, a) + d(a, y) + d(x, a) + d(a, z) + d(y, a) + d(a, z) \\ &= S(x, x, a) + S(y, y, a) + S(z, z, a). \end{aligned}$$

Lemma 2 . 5 . *In an $S-$ metric space , we have* $S(x, x, y) = S(y, y, x)$. *Proof* . By the third condition of $S-$ metric , we get

$$S(x, x, y) \leq S(x, x, x) + S(x, x, x) + S(y, y, x) = S(y, y, x) \tag{1}$$

and similarly

$$S(y, y, x) \leq S(y, y, y) + S(y, y, y) + S(x, x, y) = S(x, x, y). \tag{2}$$

Hence , by ( 1 ) and ( 2 ) , we obtain $S(x, x, y) = S(y, y, x)$.

DEFINITION 2 . 6 . Let $(X,S)$ be an $S-$ metric space . For $r>0$ and $x\in X$ we
define the open ball $B_S(x,r)$ and closed ball $B_S[x,r]$ with a center $x$ and a radius $r$ as follows :

$$B_S(x,r)=\{y\in X:S(y,y,x)<r\},$$
$$B_S[x,r]=\{y\in X:S(y,y,x)\leq r\}.$$

 EXAMPLE 2 . 7 . Let $X=\mathbb{R}$. Denote $S(x,y,z)=\mid y+z-2x\mid+\mid y-z\mid$ for all

$x,y,z\in\mathbb{R}$.Therefore

$$B_S(1,2)=\{y\in\mathbb{R}:S(y,y,1)<2\}=\{y\in\mathbb{R}:\mid y-1\mid<1\}=(0,2).$$

DEFINITION 2 . 8 . Let $(X,S)$ be an $S-$ metric space and $A\subset X$. ( 1 ) If for every $x\in A$ there exists $r>0$ such that $B_S(x,r)\subset A$, then the subset $A$ is called an open subset of $X$. ( 2 ) A subset $A$ of $X$ is said to be $S-$ *bounded* if there exists $r>0$ such that

$$S(x,x,y)<r\text{forall}x,y\in A.$$

( 3 ) A sequence $\{x_n\}$ in $X$ converges to $x$ if and only if $S(x_n,x_n,x)\to 0$ as $n\to\infty$. That is for each $\varepsilon>0$ there exists $n_0\in\mathbb{N}$ such that for all $n\geq n_0$, $S(x_n,x_n,x)<\varepsilon$ and we denote this by $\lim_{n\to\infty}x_n=x$.

( 4 ) A sequence $\{x_n\}$ in $X$ is called a *Cauchy s equence* if for each $\varepsilon>0$, there exists $n_0\in\mathbb{N}$ such that $S(x_n,x_n,x_m)<\varepsilon$ for each $n,m\geq n_0$.

( 5 ) The $S-$ metric space $(X,S)$ is said to be *complete* if every Cauchy sequence is convergent .

( 6 ) Let $\tau$ be the set of all $A\subset X$ with $x\in A$ if and only if there exists $r>0$ such that $B_S(x,r)\subset A$. Then $\tau$ is a topology on $X$( induced by the $S-$ metric $S$).

LEMMA 2 . 9 . *Let $(X,S)$ be an $S-$ metric space . If $r>0$ and $x\in X$, th en the ball $B_S(x,r)$ is an open subset of $X$.*

*Proof* . Let $y\in B_S(x,r)$, hence $S(y,y,x)<r$. If we set $\delta=S(x,x,y)$ and $r'=r-2\delta$ then we prove that $B_S(y,r')\subseteq B_S(x,r)$. Let $z\in B_S(y,r')$, therefore , $S(z,z,y)<r'$. By the third condition of $S-$ metric we have

$$S(z,z,x)\leq S(z,z,y)+S(z,z,y)+S(x,x,y)<2r'+\delta=r$$
$$\text{andso}B_S(y,r')\subseteq B_S(x,r).$$

LEMMA 2 . 1 0 . *Let $(X,S)$ be an $S-$ metric space . If the s equence $\{x_n\}$ in $X$ converges to $x$, then $x$ is unique .*

*Proof* . Let $\{x_n\}$ converges to $x$ and $y$. Then for each $\varepsilon>0$ there exist

$$n_1,n_2\in\mathbb{N}\text{suchthat}$$
$$n\geq n_1\Longrightarrow S(x_n,x_n,x)<\varepsilon 2$$

and

$$n\geq n_2\Longrightarrow S(x_n,x_n,y)<\varepsilon 2.$$

If set $n_0=\max\{n_1,n_2\}$, therefore for every $n\geq n_0$ and the third condition of $S-$ metric we get

$$S(x,x,y)\leq 2S(x,x,x_n)+S(y,y,x_n)<\varepsilon 2+\varepsilon 2=\varepsilon.$$

Hence $S(x,x,y)=0$ and so $x=y$.

Lemma 2 . 1 1 . *Let* $(X, S)$ *be an* $S-$ *metric space . If th e s equence* $\{x_n\}$ *in* $X$ *converges to* $x$, *then* $\{x_n\}$ *is a Cauchy s equence .*

*Proof* . Since $\lim_{n\to\infty} x_n = x$ then for each $\varepsilon > 0$ there exists $n_1, n_2 \in \mathbb{N}$ such that

$$n \geq n_1 \Rightarrow S(x_n, x_n, x) < \varepsilon 4$$

and

$$m \geq n_2 \Rightarrow S(x_m, x_m, x) < \varepsilon 2.$$

If we set $n_0 = \max\{n_1, n_2\}$, therefore for every $n, m \geq n_0$ we get by the third condition of $S-$ metric

$$S(x_n, x_n, x_m) \leq 2S(x_n, x_n, x) + S(x_m, x_m, x) < \varepsilon 2 + \varepsilon 2 = \varepsilon.$$

Hence , $\{x_n\}$ is a Cauchy sequence .

Lemma 2 . 1 2 . *Let* $(X, S)$ *be an* $S-$ *metric space . If there exist s equences* $\{x_n\}$ *and* $\{yn\}$ *such that* $\lim_{n\to\infty} x_n = x$ *and* $\lim_{n\to\infty} yn = y$, *th en* $\lim_{n\to\infty} S(x_n, x_n, yn) =$

$$S(x, x, y).$$

*Proof* . Since $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} yn = y$, then for each $\varepsilon > 0$ there exist $n_1, n_2 \in \mathbb{N}$ such that

$$\forall n \geq n_1, \quad S(x_n, x_n, x) < \varepsilon 4$$

and

$$\forall n \geq n_2, \quad S(yn, yn, y) < \varepsilon 4.$$

If set $n_0 = \max\{n_1, n_2\}$, therefore for every $n \geq n_0$ we get by the third condition of $S-$ metric

$$\begin{aligned} S(x_n, x_n, yn) &\leq 2S(x_n, x_n, x) + S(yn, yn, x) \\ &\leq 2S(x_n, x_n, x) + 2S(yn, yn, y) + S(x, x, y) \\ &< \varepsilon 2 + \varepsilon 2 + S(x, x, y) = \varepsilon + S(x, x, y). \end{aligned}$$

Hence we obtain

$$S(x_n, x_n, yn) - S(x, x, y) < \varepsilon \tag{3}$$

On the other hand , we get

$$\begin{aligned} S(x, x, y) &\leq 2S(x, x, x_n) + S(y, y, x_n) \\ &\leq 2S(x, x, x_n) + 2S(y, y, yn) + S(x_n, x_n, yn) \\ &< \varepsilon 2 + \varepsilon 2 + S(x_n, x_n, yn) = \varepsilon + S(x_n, x_n, yn), \end{aligned}$$

that is

$$S(x,x,y) - S(x_n,x_n,yn) < \varepsilon \tag{4}$$

Therefore by relations ( 3 ) and ( 4 ) we have $\mid S(x_n,x_n,yn) - S(x,x,y) \mid < \varepsilon$, that is

$$\lim_{n\to\infty} S(x_n,x_n,yn) = S(x,x,y).$$

DEFINITION 2 . 1 3 . Let $(X,S)$ be an $S-$ metric space . A map $F : X \to X$ is said to be a contraction if there exists a constant $0 \le L < 1$ such that

$$S(F(x),F(x),F(y)) \le LS(x,x,y), \text{forall} \quad x,y \in X.$$

**3 . A generalization of fixed point theorems in S - metric spaces**

Note that a contraction map is necessarily continuous because if $x_n \to x$ in the above condition we get $F(x_n) \to F(x)$.

For notational purposes we define $F^n(x), x \in X$ and $n \in \{0, 1, 2, ...\}$, induc - tively by $F^0(x) = x$ and $F^{n+1}(x) = F(F^n(x))$.

The first result in this section is known as a similar Banach ' s contraction principle .

THEOREM 3 . 1 . *Let* $(X, S)$ *be a complete* $S-$ *metric space and* $F : X \to X$ *be a*
*contraction . Then* $F$ *has a unique fixed point* $u \in X$. *Furthermore , for any* $x \in X$ *we have* $\lim_{n\to\infty} F^n(x) = u$ *with*

$$S(F^n(x), F^n(x), u) \le 1^{2L^n}_{-} L^{S(x,} x, F(x)).$$

*Proof* . First , we show the uniqueness . Suppose that there exist $x, y \in X$ with

$$x = F(x) \text{and} y = F(y). \text{Then}$$

$$S(x, x, y) = S(F(x), F(x), F(y)) \le LS(x, x, y)$$

$$\text{and} \therefore S(x, x, y) = 0.$$

To show the existence , we select $x \in X$ and show that $\{F^n(x)\}$ is a Cauchy sequence . For $n = 0, 1, ...,$ we get by induction

$$S(F^n(x), F^n(x), F^{n+1}(x)) \le LS(F^{n-1}(x), F^{n-1}(x), F^n(x))$$

. . .

$$\le L^n S(x, x, F(x)).$$

Thus for $m > n$ we have

$$S(F^n(x), F^n(x), F^m(x))$$

$$\le 2\sum_{i=n}^{m-2} S(F^i(x), F^i(x), F^{i+1}(x)) + S(F^{m-1}(x), F^{m-1}(x), F^m(x))$$

$$\le 2\sum_{i=n}^{m-2} L^i S(x, x, F(x)) + L^{m-1} S(x, x, F(x))$$

$$\le 2L^n S(x, x, F(x))[1 + L + L^2 + \cdots]$$

$$\le 1^{2L^n}_{-} LS(x, x, F(x)).$$

That is for $m > n$,

$$S(F^n(x), F^n(x), F^m(x)) \le 1^{2L^n}_{-} LS(x, x, F(x)). \tag{5}$$

This shows that $\{F^n(x)\}$ is a Cauchy sequence and since $X$ is complete there exists $u \in X$ with $\lim_{n\to\infty} F^n(x) = u$. Moreover , the continuity of $F$ yields

$$u = \lim_{n\to\infty} F^{n+1}(x) = \lim_{n\to\infty} F(F^n(x)) = Fu.$$

 Therefore , $u$ is a fixed point of $F$. Finally letting $m \to \infty$ in ( 5 ) we obtain

$$S(F^n(x), F^n(x), u) \leq 1^{2}{}_{-}^{L^n} L^{S(x,} x, F(x)).$$

EXAMPLE 3 . 2 . Let $X = R$, then $S(x, y, z) =| x - z | + | y - z |$ is an $S-$ metric on $X$. Define a self - map $F$ on $X$ by : $F(x) = 2^1 \sin x$. We have

$$S(Fx, Fx, Fy) =| 2^1( \sin x - \sin y) | + | 2^1( \sin x - \sin y) |$$

$$\leq 2^1(| x - y | + | x - y |) = 2^{1S(x,} x, y)$$

for every $x, y \in X$. Furthermore , for any $x \in X$ we have $\lim_{n\to\infty} F^n(x) = 0$ with

$$S(F^n(x), F^n(x), 0) \leq 1^{2}{}_{-}^{L^n} L^{S(x,} x, F(x)), L = 2^1.$$

It follows that all conditions of Theorem 3 . 1 hold and there exists $u = 0 \in X$ such

$$\text{that} u = Fu.$$

THEOREM 3 . 3 . *Let* $(X, S)$ *be a compact* $S-$ *metric space with* $F \quad : X \to X$ *satisfying* $S(F(x), F(x), F(y)) < S(x, x, y)$ *for al l* $x, y \in X$ *and* $x \neq y$. *Then* $F$ *has a unique fixed point in* $X$.

*Proof* . The uniqueness part is easy . To show the existence , notice that the map $x \mapsto S(x, x, F(x))$ attains its minimum , say at $x_0 \in X$. We have $x_0 = F(x_0)$ since otherwise

$$S(F(F(x_0)), F(F(x_0)), F(x_0)) < S(F(x_0), F(x_0), x_0) = S(x_0, x_0, F(x_0))$$

which is a contradiction .

Next , we present a lo cal version of Banach ' s contraction principle . THEOREM 3 . 4 . *Let* $(X, S)$ *be a complete* $S-$ *metric space and le t*

$$B_S(x_0, r) = \{x \in X : S(x, x, x_0) < r\}, where \quad x_0 \in X and \quad r > 0.$$

*Suppose that* $F : B_S(x_0, r) \to X$ *is a contraction with*

$$S(F(x_0), F(x_0), x_0) < (1 - L)r2.$$

*Then* $F$ *has a unique fixed point in* $B_S(x_0, r)$.

*Proof* . There exists $r_0$ with $0 \leq r_0 < r$ such that $S(F(x_0), F(x_0), x_0) \leq (1 - L)r_0 2$. We will show that $F : B_S(x_0, r_0) \to B_S(x_0, r_0)$. To see this , note that

$$\text{if} x \in B_S(x_0, r_0), \text{then}$$
$$S(x_0, x_0, F(x)) \leq 2S(x_0, x_0, F(x_0)) + S(F(x_0), F(x_0), F(x))$$
$$\leq 2(1 - L)r_0 2 + LS(x_0, x_0, x) \leq r_0.$$

 We can now apply Theorem 3 . 1 to deduce that $F$ has a unique fixed point in $B_S(x_0, r_0) \subset B_S(x_0, r)$. Again , it is easy to see that $F$ has only one fixed point in

$$B_S(x_0, r).$$

Next , we examine briefly the behavior of a contractive map defined on $B_S(r) = B_S(0, r)$( the closed ball of radius $r$ with centre 0 ) with values in Banach space $E$. More general results will be presented in the next theorem .

Theorem 3 . 5 . *Let $(X, S)$ be a complete $S-$ metric space with $S(x, y, z) = \| x - y \| + \| y - z \|$ and le t $B_S(r)$ be the clos ed ball of radius $r > 0$, central at zero in Banach space $E$ with $F : B_S(r) \to E$ a contraction and $F(\partial B_S(r)) \subseteq B_S(r)$. Then $F$ has a unique fixed point in $B_S(r)$.*

*Proof* . Consider $G(x) = x + 2^{F(x)}$. We first show that $G : B_S(r) \to B_S(r)$. To see this , let

$$x^* = r \| x^x \| \quad \text{where} \quad x \in B_S(r) \text{and} \quad x \neq 0.$$

Now if $x \in B_S(r)$ and $x \neq 0$, we have

$$\begin{aligned} S(F(x), F(x), F(x^*)) = \quad & \| F(x) - F(x^*) \| \leq LS(x, x, x^*) = L \quad \| x - x^* \| \\ & = L \quad \| x - r \| x^x \| \quad \| = L(r - \| x \|) \end{aligned}$$

Hence

$$\| F(x) \| \quad \leq \quad \| F(x^*) \| \quad + \quad \| F(x) - F(x^*) \| \leq r + L(r - \quad \| x \|) < 2r - \| x \|$$

Then for $x \in B_S(r)$ and $x \neq 0$

$$\| G(x) \| \quad = \| x + 2^{F(x)} \| \leq \quad \| x \| + \|_2^{F(x)} \| \quad \leq r.$$

In fact by the continuity of $G$ we get $\| G(0) \| \leq r$, and consequently $G : B_S(r) \to B_S(r)$. Moreover $G : B_S(r) \to B_S(r)$ is a contraction because

$$\| G(x) - G(y) \| \leq \quad \| x - y \| \quad + 2L \| x - y \| = (1 + 2L) \| x - y \| .$$

Theorem 3 . 1 implies that $G$ has a unique fixed point in $u \in B_S(r)$ and so $u = Fu$.

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