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Some Properties of $\theta-$ open Sets

Algunas Propiedades de los Conjuntos $\theta-$ abiertos

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Abstract

In the present paper , we introduce and study topological properties of $\theta-$ derived , $\theta-$ border , $\theta-$ frontier and $\theta-$ exterior of a set using the con - cept of $\theta-$ open sets and study also other properties of the well known notions of $\theta-$ closure and $\theta-$ interior .

Key words and phrases : $\theta-$ open , $\theta-$ closure , $\theta-$ interior , $\theta-$ border , $\theta-$ frontier , $\theta-$ exterior .

Resumen

En el presente ert í culo se introducen y estudian las propiedades to - pol ó gicas del $\theta-$ derivedo , $\theta-$ borde , $\theta-$ frontera y $\theta-$ exterior de un conjunto usando el concepto de conj unto $\theta-$ abierto y estudiando tambi é n otras propiedades de las nociones bien conocidas de $\theta-$ clausura y $\theta-$ interior .

Palabras y frases clave : $\theta-$ abierto , $\theta-$ clausura , $\theta-$ interior , $\theta-$ borde ,

$$\theta - \text{frontera}, \theta - \text{exterior}.$$

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1 Introduction

The notions of $\theta-$ open subsets , $\theta-$ closed subsets and $\theta-$ closure where introduced by Veli č ko [14] for the purpose of studying the important class of H - closed spaces in terms of arbitrary fiberbases . Dickman and Porter [2] , [3] , Joseph [9] and Long and Herrington [1 1] continued the work of Veli č ko . Recently Noiri and Jafari [1 2] and Jafari [6] have also obtained several new and interesting results related to these sets . For these sets , we introduce the notions of $\theta-$

derived , $\theta-$ border , $\theta-$ frontier and $\theta-$ exterior of a set and show that some of their properties are analogous to those for open sets . Also , we give some additional properties of $\theta-$ closure and $\theta-$ interior of a set due to Veli č ko [1 4] . In what follows $(X,\tau)($ or $X)$ denotes topological spaces . We denote the interior and the closure of a subset A of X by Int (A) and $Cl(A)$, respectively . A point $x \in X$ is called a $\theta-$ adherent point of A [14], if $A\cap notdef-parenleftnotdef-V)^{notdef-notdef} \neq$ $emptyset-notdefnotdef-f$ r $notdefnotdef - notdef - enotdef - enotdef - r$ y $negationslash - notdef - notdef\{$ en s t V c ntaining x T h − e s t o al l θ ad herent p ints o As − i c l − l e v dt e c − l osure o A o a d s − i d noted b y $Cl(A)$.$-_{\mathrm{A}}$ s bset A o X i − s c l − l ed θc − l osed A $= Cl(A)$.$\mathrm{n_D}$ o − n tchev a d M k − $\mathrm{a_i}$[[], L emma 3nine − bracketright h f ve s own t atif A d B a esu bsets o a s − p ace $(X\tau)$, t h en $Cl(A \cup B) = Cl(A) \cup$ $Cl(B)$ a d $\theta(A \cap notdef$equal − notdef − notdef$Cnotdef_\theta A - notdef)notdefintersection -$ $notdef$ $theta-notdef-notdef-notdefnotdef-notdef-B$period − notdef − notdef − negationslash − notd $braceleftnotdef-notdef-notdef-N^{notdef-o-notdef-t}$ e $\in$ s − notdef − notdef − notdef − notdef − l_{notdef} $notdef-negationslash-notdef-a\{notdef-notdef-tnotdefnotdef-helement-$ $e_{notdef-thetanotdef-notdef}$c − notdef − notdef$notdef-o_{union-s}notdef-r_{enotdef}\theta_{notdef-\mathrm{ofa}}$ g ve n set ot b a θhyphen − $\mathrm{c_l}$ s d se period − t B ut i isa lw$^{\mathrm{a}}$ ay c l − o sed . Dick m an a d − n P o r − t e [2]

dthata com p ctsubsp ce $\mathrm{t_{ofa}}$ H au s − dr − $\mathrm{o_{ff}}$ space i − s $\theta-$ c os d . M or e e − $\mathrm{v_{r,}}$

howe dthata $\theta-$ cl s d s a ubsp ce ofa H au d o − $\mathrm{r_{ff}}$ space s l − c os ed . Jank o v − $\mathrm{i}_{\acute{c}}$ ve dthata spac e (X,τ) isH au sd o − $\mathrm{r_{fff-i}}$ and on lyi ev $\mathrm{i_e}$ y com p cts e − $\mathrm{t^{s-i}}$ d . The c m p eme n − t ofa θ c o − l s d set is all ed a $\theta-$ op e nse t − period T he f miy

- o pen se t − so − f rm s a t o − p o o − l gy on X an i den ot d by τ. T is t o − p olo gy sr e rth an τ and i is ew l - k n − o w n th t − a a s spa e (X,τ) isr eg lu a rifa nd

$\tau = \tau\theta$. tI s a so $\overset{\mathrm{t}}{\underset{\mathrm{o}}{}}$ $bvio$ sth ata set A s θ c os d i n (X,τ) ifa nd on ly c os ed i n $(X,\tau\theta$.

that a poi t $x \in X$ i s a − l ed the δ cus er point f − o $A \subseteq$ X $A_{\mathrm{f}} \cap notdef(notdef - C$parenleft − notdef a l n l t cl ste rp

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$\alpha-$ closed) . The intersection of all semi - closed (resp . $\alpha-$ closed) sets contain - ing A is called the semi - closure (resp . $\alpha-$ closure) of A and is denoted by $sCl(A)($ resp . $\alpha Cl(A))$. Recall also that a space (X,τ) is called extremally disconnected if the closure of each open set is open . Ganster et al . [[5] , Lemma 0 . 3] have shown that For $A \subset X$, we have $A \subseteq sCl(A) \subseteq Cl_{\theta}(A)$ and also if (X,τ) is extremally disconnected and A is a semi - open set in X, then $sCl(A) = Cl(A) = Cl_{\theta}(A)$. Moreover , it is well - known that if a set is preopen , then the concepts of $\alpha-$ closure , $\delta-$ closure , closure and $\theta-$ closure coincide . In [1 3] , M . Steiner has obtained some results concerning some characterizations of some generalizations of T_1 spaces by utilizing $\theta-$ open and $\delta-$ open sets . Also , quite recently Cao et al . [1] obtained , among others , some substantial results concerning the $\theta-$ closure operator and the related notions . In general , we do

not know much about $\theta-$ open sets and dealing with them are very difficult .

2 Properties of $\theta-$ open Sets

Definition 1 . Let A be a subset of a space X. A point $x \in X$ is said to be $\theta-$ limit point of A if for each $\theta-$ open set U containing $x, U \cap notdef^{backslash-braceleft-notdef} xbraceright-notdefnotdef-parenrightnotdef \neq emptyset - notdef - notdef - notdefnotdefnotdef$ h $-$ e e $-$ s tof al l $\theta - $l $-$ i mitpo intsof A s $-$ i ca le d t $-$ h e θ derived e $-$ s tof A a dis de noted

y$D(A)$.

horem 2 1 . *F rsu bsets A, B o a s $-$ p ace X, h efo l $-$ l o wing s $-$ t a te m n $-$ e ts h l d $-$ colon)*^{e}Dparenleft $-$ A) $\subset D(A$ *o) w ere $DA - parenleft)$ is fth e d r i $-$ v ed e $-$ s t o f A .) If A* $\subset B$ *th en* $D(A) \subset D(B$parenright $-$ period

$D(B) = D(A \cup B) ad \quad D(A \cap notdef - parenrightnotdef - propersubsetnotdef - Dnotdefnotdef - A_{notdef}$)$D(A) \cup$

$(A$

(1) It s u ffi ce stoo b er e thate very $\theta-$ ope nsetis ope n . $\theta(A \cup B) = D\theta A(\cup \quad D(B$ is am o i c $-$ a tion of t e $-$ h st and ardproof f $-$ o r e $-$ r$_{e}$ open etsa rere pac ed by $\theta-$ pe nset period $-$ s

$x \in D \ D \ (A)^{\text{parenright}-\text{backslash}} A^{l}$ a n d U is a - o e n s t $-$ e c nta ning x, th n $U \cap D - notdef$ parenleft $-$ notdefparenright $-$ notdef

$\theta($ θ θ p i y $\in U, U\backslash$

$x \in A$.Hnce

$D)_{\theta(}$Aparenright − backslash$A$, thn

T − h$_{\mathrm{u}}$s

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f − rom

4) $notdef$ tha U $\cap$ $Abackslash - notdefbraceleft - notdef_x$

(

$$D_\theta(A \cup D_\theta(A)) \subset A \cup D_\theta(A).$$

In general the equality of (1) and (3) does not hold .

Example 2 . 2 . (i) Let $X = I \times 2$ has the product topology , where $I =< 0,1 >$ has the Euclidean topology and $2 = \{0,1\}$ has the Serpi ń ski topology with the singleton { 0 } open . Then $A \subset X$ is $\theta-$ closed ($\theta-$ open , respectively) if and only if $A = B \times 2$, where $B \subset I$ is closed (open , respectively) .

Observe that if $A \subset X$ is $\theta-$ closed , then $Cl_\theta(A) = A$. Let $B = \pi_I(A) \subset I$. Obviously , $A \subset B \times 2$. Let $(x, y) \in B \times 2$. Then $x \in B$, so there is some $(x', y') \in A$, such that $\pi_I(x', y') = x$. Hence $x' = x$, so $(x, y') \in A$. Let H be a closed neighborhood of (x, y). Then H contains both of the points $(x, 0)$, $(x, 1)$ and so H contains (x, y') as well . It follows that $H \cap notdef-negationslash-equalnotdef-emptyset-notdefnotdef-a$d − notdef$notdef$t − notdef e n − notdef$_{,notdef-notdefnotdef}$
$, y) \in Cl(A) = A$. H e − n$_{ce,}$$A = B \times 2$ L t $z \in I \setminus B$. T e − h n
$(,0)$ $element-slashA$. S nce i − s$\theta-$ cl osed , t h ere e ist $\varepsilon > 0$ s ch t at
$(< z-, z+\varepsilon >$ ×parenright − two $\cap$ $notdef-equalemptyset-notdefnotdef$
T $notdef - h_{notdef} notdef - n$ $notdefnotdef - notdef$

$-\varepsilon z + \varepsilon > \cap notdef - equalnotdefnotdef - comma - notdefnotdef$w$notdef - i - c_h notdef$m − notdef$_{enotde}$

$A = \times\{1$. T h e n $D\theta(A) = X$ ub t $(A) =$ c A. H n c e $D\theta(A) \not\subset D(A$.
c o n erexam p l − el − i ust rating t hat
$D\theta(A\cap) notdef^{equal-negationslash-notdef} notdefnotdef$(parenright − notdef$intersection-notdef-notdef_{\text{parenleft−notdef−notdef}} notdef-B^{notdef-notdef} notdefnotdef-notdef-negationslash-n-notdefnotdef-notdef-notdef_{notdef} notdef^{notdef-notdef} notdefnotdef-notdefnotdef - notdef - notdef$
n be ea s − i ly ou ndin egula s aces (e g . in **R**) , f o D_{w}i − h c h op n
) $\overset{notdef}{\text{a}}$ nd $notdef$
s (and henc e D nd D) oin de . .
.three − period Let $(Z, \mathcal{K})$ be the dg t − i$_a$ ln - pa ce − t h edigt al l − i$_n$ eor the soc al ed yine . Thi ist h e s − e t of the int ge r comma − s Z, e quip p e − d withthet opol gy t − e$_d$ by : s

1} : $n \in Z\}$.The − n[4 : f − I $A = \{x\}$ $n-1, 2n, 2+$

$) \neq$ $Cl(A)$ f is ev n .) $= Cl(A)$ i f x is od d .

$) \subset C_\theta(A)$. **mtwo − period**$_{4.}$ $A \cup D_\theta(A$
$A) \subset$ $C_\theta(A) A$ $\cup D\theta(A) \subset C_\theta(A)$. ce$D\theta$

y 2.**five − period** $f - IAi - s\ a$ - $c\ o\ ed\ s\ ub\ set$, $th\ enitc\ o\ n\ \ a - t_{in}\ st$
heset of $i - t\ s\ \ \theta - \ l\ i\ m\ \ t - i$

n 2 . A po t x X is s i − d t b ea θ in er r p oint of A if t er e op ens et U co tai i − n g x su chth a $U \subset$ $Cl() \subset A$. Th s t f l − a_l p oints of A is n_s id t − o b ethe $\theta-$ nt r o − i_r of $AU[9]$ a nd i de n o − t e db y

ou st hat ano pen et U in X $\theta-$ o e − nf − i$Int - theta(U)$ $=$ $U[^{\text{one}-\text{bracketleft}}1$,
1] .

 Theorem 2 . 6 . *For subsets* A, B *of a space* X, *th e following s tatements are*

true :

(1)$Int_{\theta}(A)$ *is the union of al l open s e ts of* X *whose c losures are contained in* A. (2)A *is* $\theta-$ *open if and only if* $A = Int_{\theta}(A)$.

(8)$Int_{\theta}(A) \cap notdefnotdef(B - notdefnotdef^{notdef-parenright} = notdefntnotdef(notdef - notdef - Anotd$

f.(5)

ni t $-$ i$_{\mathbf{on}}$ 3.$\theta_{(}$ $A) = A \setminus In\theta(A)$ iss ia d tob et he $\theta-$ b r e $-$ d$_{\mathrm{r}}$ of A. **orem 2period $-$ seven** *F o r a u $-$ s b et* A *o a sp a e* X, *he oll owing s t $-$ a t e me n s $-$ t hol :*
$b(A) \subset b(A)$ *whe e b $-$ parenleftA) den testhe bor er of* A.

$A = Itn(A) \cup b\theta()$.
$Int(A) \cap (notdefnotdef - parenright = notdefemptyset - notdef.notdefnotdefnotdefnotdef - notdef$

isa θ *op e n s etif a n d o n$-$l yi* $b\theta(notdefA)notdef_{=}notdef^{period-emptyset}notdef$ $notdef$

$t\theta($

parenleft $-$ b$\theta(A))$

$(A) = A \cap \theta_{notdef} notdef - Xnotdef - backslashA - notdef - parenright.notdefnotdefnotdefnotdef - not$

5) If $\in$ $In(bA))$, th en x $\in$ $b(A$. O nth oh r h and , s nce
$(_{A}, x \in In\theta bA))$ $\subset Int\theta A$. H e nce $x \in In\theta A \mathrm{e}_{\cap notdef-parenleft_{A}} notdef^{notdef-parenright}$w $-$ notdef$notdef-$
$hnotdef - \mathrm{c_h} notdefnotdef$ $notdef - notdefnotdefnotdefnegationslash - notdef -$
$notdef$ s (3) . T hus $In(_{t}$parenleft $-$ b$p(A$parenright $-$ parenright $= \varnothing$.)

$$notdefnotdefnegationslash - notdef - notdef\{$$
$$^{\theta}\ A \setminus (X \setminus l_{\theta}(X \setminus A) = A \cap notdef(notdef \setminus notdefA - notdef\text{period} - \text{notdef}notdefnotdefnotdef - notdef$$

. Let $X = \{a$ b, c w ht $\tau = \{\varnothing\{a\}\{\}, \{a, b\}, X\}$. Th e ni t ca n ifi d th a for $,_{A} = \}\{b\}$, $(A) propersubset - negationslash b(A)$, i $-$ period e i n gen e a l e qu lty of Th$^{\mathrm{o-e}}$ re m 2.(one $-$ parenright does n o t

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four − period $Fr_\theta(A) = C \quad l(A) \setminus Int(A)$s − is − a id t obe t e θ− ro ti r [6 o f A. **period − nine**. *Fora u b s e* A *o f asp a c* X, *t h efolo i* − w_n *gst te mentsh o ld :*
$\subset Fr(A)$ *w h − e e* $Fr(A)$ *d eno es the rontier of* A.

$\cap notdef \theta notdef - parenleft_{notdef-A)notdef}$equal − notdef$notdef \varnothing notdef notdef notdef - notdef notdef notdef$

$notdef notdef - notdef notdef notdef not$

$Cl\theta A) \cap notdef \theta X notdef backslash - notdef - A$parenright − notdef$notdef - period notdef$

$$(6) Fr_\theta(A) = Fr_\theta(X \setminus A).$$

$(7) Fr_\theta(A)$ *is clos ed .*

Proof.

$(3) Int_\theta(A) \cap notdefnotdef - parenleft_A notdefnotdefnotdef - equalnInotdef - t_{notdef(notdef-notdef-A)}no$

$\setminus Fr(A) = A \setminus C($

er l , the e q ual tesi n (1) an d () o − f the T heo em 2 . 9 d n o th lda s ho^{g}nwn byth e f − ol − l ow ing examp le . 4 $p - l_{e2one-period0}$. C o n s d − e_{rth} eto pol gicalspa ce (X, τ) i − g_v en in Ex am p − $l_{e2eight-period.}$ I

$b - braceright$. T he n $Fr_\theta() = b - braceleft, \}_c \not\subset^o \{c\} =$ $FrA($ an da so

$Fr\theta() = braceleft - b, \}_c \not\subset^f$

$b\theta($ A $)$

k two − period11. L e t A and i B s ub e − s s o − fX. Th n − e $A \subset B$ d o s otim ply that F $r(B) \subset Fr()$ or $Fr(A) \subset Fr()$. T h − e r ade rcan b eve f − i_y this

θ y − period

i − t_{o-i} n 5. Et_θ $A) = Int\theta_($X \setminus A)$^{i-s}$ sai dt ob e be a θ − x − e t riorof A.

r − e m 2 . 12 *F or a sub e A o f a s ac e X, th fo low ing sta e − t m e ntsho l − d :* $xt\theta A) \subset Ex()$ *w* h^{e-r} *e* $Ext(A)$ *de o − n t s t − h e* e_{ex} *te i − r orof A.* $xt\theta A)$ *is o − p e n .*

$xt\theta A) = Int(_\theta X \setminus A) =$

$xt\theta A\cap)notdef^{propersuperset-notdef}notdef - Ex - notdef - tnotdef - thetanotdef - Aintersection - notdef$

$X)(= \varnothing.$

$Int\theta A)\cup$

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$t_\theta(X \setminus A)) \subset Int(X$

$nA) \subset .Int\theta(C(A)) = Int(XI \setminus nt(X \setminus A)$parenright − parenright $= Int(X$

$(A)).$

3 Aplications of $\theta-$ open Sets

Definition 6 . Let X be a topological space . A set $A \subset X$ is said to be $\theta-$ saturated if for every $x \in A$ it follows $Cl_\theta(\{x\}) \subset A$. The set of all $\theta-$ saturated sets in X we denote by $B_\theta(X)$.

Theorem 3 . 1 . *Let X be a topological space . Then $B_\theta(X)$ is a complete Boolean s e t alg e bra .*

Proof . We will prove that all the unions and complements of elements of $B_\theta(X)$ are members of $B_\theta(X)$. Obviously , only the proof regarding the com - plements is not trivial . Let $A \in B_\theta(X)$ and suppose that $Cl_\theta(\{x\}) \not\subset X \setminus A$ for some $x \in X \setminus A$. Then there exists $y \in A$ such that $y \in Cl_\theta(\{x\})$. It follows that x, y have no disj oint neighbourhoods . Then $x \in Cl_\theta(\{y\})$. But this is a contradiction , because by the definition of $B_\theta(X)$ we have $Cl_\theta(\{y\}) \subset A$. Hence $, Cl_\theta(\{x\}) \subset X \setminus A$ for every $x \in X \setminus A$, which implies $X \setminus A \in B_\theta(X)$.

Corollary 3.2. *$B_\theta(X)$ contains every union and every inters ectio n of $\theta-$ c los ed and $\theta-$ open s e ts in X.*

A filter base Φ in X has a $\theta-$ cluster point $x \in X$ if $x \in \cap \quad Cl_\theta(F) \mid F \in \Phi\}$. The filter base $\Phi\theta-$ converges to its $\theta-$ limit x if for every closed neighbourhood H of x there is $F \in \Phi$ such that $F \subset H$. A net $f(B, \geq)$ has a $\theta-$ cluster point (a $\theta-$ limit $)x \in X$ if x is a $\theta-$ cluster point (a $\theta-$ limit) of the derived filter base

$$\{f(\alpha) \mid \alpha \geq \beta \mid \beta \in B\}.$$

Recall that a topological space X is said to be (countably $)\theta-$ regular [5] , [7] if every (countable) filter base in X with a $\theta-$ cluster point has a cluster point . Obviously , a space X is $\theta-$ regular if and only if every $\theta-$ convergent net in X has a cluster point .

Theorem 3 . 3 . *Let X be a $\theta-$ regular topological space . Then every e lement of $B_\theta(X)$ is $\theta-$ regular .*

Proof . Let $f(B, \geq)$ be a net in $Y \in B_\theta(X)$, which $\theta-$ converges to $y \in Y$ in the topology of Y. Then $f(B, \geq)\theta-$ converges to y in X and hence $, f(B, \geq)$ has a cluster point $x \in X$. One can easily check that x, y have no disj oint neighbourhoods in X, which implies that $x \in \quad Cl_\theta(\{y\})$ and hence $x \in Y$. Then every $\theta-$ convergent net in Y has a cluster point in Y, which implies that

$$Y \text{is} \theta - \text{regular}.$$

Recall that a subspace of a topological space is θF_σ if it is a union of countably many $\theta-$ closed sets . A subspace of a topological space called θG_δ if it is an intersection of countably many $\theta-$ open sets .

Example 3 . 4 . There is a compact topological space X containing an $F_\sigma-$ subspace Y which even is not countably $\theta-$ regular .

Proof . Let $Y = \{2, 3, ...\}, U_x = \{n \cdot x \mid n = 1, 2, ...\}$ for every $x \in Y$. The family $S = \{U_x : x \in Y\}$ defines a topology (as its base) on Y. Since $U_x \cap notdef \neq notdef$

f r e v − e ry $x, y \in Y$, e e − v r yo penn one mp t − y s t $U \subset Y$ h as $ClYU = Y$. t llo wst h a − t t hen t − e$i(dP, \geq$ parenright − comma w h r − e$_e P$ i t hes t o

fa l p r − i men umb $r - e$ I swi − th

h e − i$_r$ n t − a u r − a l o rd r − e $\geq$, i c early θ c on v − e rge n comma − t b utw i − t h l n oc uster p on ti n Y. tf llo wst hat Y i n otc ouna by θ reg uar . L e − t$X = \{1\} \cup Y$ a ndt akeo n t het opoo gyo fA l − e xand o − rffquoteright − s^l_compa^{t-c} ifi c − a$_{tio}$ no f Y. T os e t h t − aY i a n - s ubspa c − e o f X, l t $Kx = Y \setminus \bigcup y > x^U y_f$ ore e − v r y $x \in Y$. E v e − r y Kx_i c osed ,

nte , a ndh en c − e c ompa^{t-c} i nt opoo gyo f Y. I tf llo wst h a − tKx_i c osed i n X.

$$\mathrm{n^ic-e}x \in K, Y =^{\bigcup} x - infinity = 2K.$$

oro l − l a ry 3 5 . *I nc ntrast t o F− s b − u s paces , e ery θF− s u bspace o a θ re gular a c − e i θ r e gular .*

oro l − l a ry 3 6 . *E e − $v_{ry}\theta G-$ s u bspace o a θ r e gular s ace i θ r e gular .*

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