

BOUNDARY VALUE PROBLEMS FOR DIFFERENTIAL EQUATIONS ORDER WITH FRACTIONAL

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Abstract . In this paper , we shall establish sufficient conditions for the existence of solutions for a first order boundary value problem for fractional differential equations .

Full t e x t

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